



Inductive Sequences and Sigma Notation Exam Questions Sheet 2

Q1.

A sequence u_1, u_2, u_3, \dots satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geq 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4 ,

(2)

(b) evaluate

$$\sum_{r=1}^4 u_r$$

(3)

(Total 5 marks)

Q2.

A sequence of numbers $a_1, a_2, a_3 \dots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \quad (n \geq 1)$$

where c is a constant.

(a) Write down an expression, in terms of c , for a_2

(1)

(b) Show that $a_3 = 12 - 3c$

(2)

Given that

$$\sum_{i=1}^4 a_i \geq 23$$

(c) find the range of values of c

(4)

(Total 7 marks)



Q3.

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = k,$$

$$a_{n+1} = 5a_n + 3, \quad n \geq 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k .

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^4 a_r$ in terms of k , in its simplest form.

(ii) Show that $\sum_{r=1}^4 a_r$ is divisible by 6.

(4)

(Total 7 marks)

Q4.

A sequence a^1, a^2, a^3, \dots is defined by

$$a_1 = 2$$

$$a_{n+1} = 3a_n - c$$

where c is a constant.

(a) Find an expression for a_2 in terms of c .

(1)

Given that $\sum_{i=1}^3 a_i = 0$

(b) find the value of c .

(4)

(Total 5 marks)



Q5.

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = k,$$
$$a_{n+1} = 3a_n + 5, \quad n \geq 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k .

(1)

(b) Show that $a_3 = 9k + 20$.

(2)

(c) (i) Find $\sum_{r=1}^4 a_r$ in terms of k .

(ii) Show that $\sum_{r=1}^4 a_r$ is divisible by 10.

(4)

(Total 7 marks)

Q6.

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = k,$$
$$a_{n+1} = 2a_n - 7, \quad n \geq 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k .

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^4 a_r = 43$,

(c) find the value of k .

(4)

(Total 7 marks)



Q7.

A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \quad n \geq 1,$$
$$a_1 = 2$$

(a) Find a_2 and a_3 , leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

(Total 4 marks)

Q8.

A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(a) Find x_2 in terms of p .

(1)

(b) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(c) find the value of p ,

(3)

(d) write down the value of x_{2008} .

(2)

(Total 8 marks)



Q9.

A sequence of numbers a_1, a_2, a_3, \dots is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

(Total for question = 7 marks)

Q10.

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \quad n \geq 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k , giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^3 a_r = 10$

(b) find an exact value for k .

(3)

(Total for question = 6 marks)

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Q11.

A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \quad n \geq 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k , giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^3 a_r = 10$

(b) find an exact value for k .

(3)

(Total for question = 6 marks)

Q12.

A sequence x_1, x_2, x_3, \dots is defined by

$$x_1 = 1,$$
$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a .

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a .

(3)

(Total for question = 6 marks)