Inductive Sequences and Sigma Notation Exam Questions Sheet 2

Q1.

Question Number	Scheme		Marks
	$u_2 = 9,\ u_{n+1} = 2u_n - 1,\ n \geqslant 1$		
(a)	$u_3 = 2u_2 - 1 = 2(9) - 1 \quad (=17)$	$u_3 = 2(9) - 1.$	M1
	$u_4 = 2u_3 - 1 = 2(17) - 1 = 33$	Can be implied by $u_3 = 17$	
		Both $u_3 = 17$ and $u_4 = 33$	A1
			[2]
(b)	$\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$		
	$(u_1) = 5$	$(u_1) = 5$	B1
	$\sum_{r=1}^{4} u_r = "5" + 9 + "17" + "33" = 64$	Adds their first four terms obtained legitimately (see notes below)	M1
		64	A1
			[3]
			5 marks
	Notes		
(a)	**M1**: Substitutes 9 into RHS of iteration formula **A1**: Needs **both** 17 and 33 (but **allow if either or both seen in part (b)**)		
(b)	**B1: for** u_1=5 (however obtained – may appear in (a)) May be called a=5 **M1**: Uses their u_1 found from $u_2 = 2u_1 - 1$ stated explicitly, or uses $u_1 = 4$ *or* $5\frac{1}{2}$, and adds it to u_2, their u_3 and their u_4 only. (See special cases below). There should be no fifth term included. Use of sum of AP is irrelevant and scores M0 **A1**: 64		

Q2.

Question Number	Scheme	Marks
	$a_1 = 3,\ a_{n+1} = 2a_n - c,\ \ n \geq 1$, c is a constant	
(a)	$\{a_2 =\}\ 2\times 3 - c$ or $2(3) - c$ or $6 - c$	B1
		[1]
(b)	$\{a_3 =\}\ 2\times(\text{"}6-c\text{"}) - c$	M1
	$= 12 - 3c$ (*)	A1 cso
		[2]
(c)	$a_4 = 2\times(\text{"}12-3c\text{"}) - c \qquad \{= 24 - 7c\}$	M1
	$\left\{\sum_{i=1}^{4} a_i =\right\}\ 3 + (6-c) + (12-3c) + (24-7c)$	M1
	$\text{"}45 - 11c\text{"} \geq 23$ or $\text{"}45 - 11c\text{"} = 23$	M1
	$c \leq 2$ or $2 \geq c$	A1 cso
		[4]
		7
	Notes	
(a)	The answer to part (a) cannot be recovered from candidate's working in part (b) or part (c). Once the candidate has achieved the correct result you can ignore subsequent working in this part.	
(b)	**M1:** For a correct substitution of *their* a_2 *which must include term(s) in c* into $2a_2 - c$ giving a result for a_3 in terms of only c. Candidates must use correct bracketing for this mark. **A1:** for correct solution only. No incorrect working/statements seen. (**Note:** the answer is given!)	
(c)	**1st M1:** For a correct substitution of a_3 *which must include term(s) in c* into $2a_3 - c$ giving a result for a_4 in terms of only c. Candidates must use correct bracketing (can be implied) for this mark. **2nd M1:** for an attempt to sum their a_1, a_2, a_3 and a_4 only. **3rd M1:** for their sum (of 3 or 4 or 5 consecutive terms) $=$ or $\geq$ or $>$ 23 to form a linear inequality or equation in c. **A1:** for $c \leq 2$ or $2 \geq c$ from a correct solution only.	
	Beware: $-11c \geq -22 \Rightarrow c \geq 2$ is A0. **Note:** $45 - 11c \geq 23 \Rightarrow -11c \leq -22 \Rightarrow c \leq 2$ would be A0 cso.	
	Note: Applying either $S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + l)$ is 2nd M0, 3rd M0.	
	Note: If a candidate gives a numerical answer in part (a); they will then get M0A0 in part (b); but if they use the printed result of $a_3 = 12 - 3c$ they could potentially get M0M1M1A0 in part (c)	
	Note: If a candidate only adds numerical values (not in terms of c) in part (c) then they could potentially get only M0M0M1A0.	
	Note: For the 3rd M1 candidates will usually sum a_1, a_2, a_3 and a_4 or a_2, a_3 and a_4 or a_2, a_3, a_4 and a_5 or a_1, a_2, a_3, a_4 and a_5	

Q3.

Question Number	Scheme	Marks
(a)	$(a_2 =)\ 5k+3$	B1 (1)
(b)	$(a_3 =)\ 5(5k+3)+3$ $= 25k+18$ (*)	M1 A1 cso (2)
(c) **(i)** **(ii)**	$a_4 = 5(25k+18)+3 \quad (=125k+93)$ $\sum_{r=1}^{4} a_r = k+(5k+3)+(25k+18)+(125k+93)$ $= 156k+114$ $= 6(26k+19)$ (or explain each term is divisible by 6)	M1 M A ao A (4) 7
	Notes (a) $5k+3$ must be seen in (a) to gain the mark (b) 1st M: Substitutes their a_2 into $5a_2+3$ - note the answer is given so working must be seen. (c) 1st M1: Substitutes their a_3 into $5a_3+3$ or uses $125k+93$ 2nd M1: for **their** sum $k+a_2+a_3+a_4$ - must see evidence of **four terms with plus signs and must not be sum of AP** 1st A1: All correct so far 2nd A1ft: Limited ft – previous answer must be divisible by 6 (eg $156k+42$) . This is dependent on second M mark in (c) Allow $\frac{156k+114}{6} = 26k+19$ without explanation. No conclusion is needed.	

Q4.

Question Number	Scheme	Marks
(a)	$(a_2 =)\ 6-c$	B1 (1)
(b)	$a_3 = 3(\text{their } a_2) - c \quad (= 18 - 4c)$ $a_1 + a_2 + a_3 = 2 + "(6-c)" + "(18-4c)"$ $"26-5c" = 0$ So $c = 5.2$	M1 M1 A1ft A1 o.a.e (4) 5
	Notes	
(b)	1st M1 for attempting a_3. Can follow through their answer to (a) but it must be an expression in c. 2nd M1 for an attempt to find the sum $a_1 + a_2 + a_3$ must see evidence of sum 1st A1ft for their sum put equal to 0. Follow through their values but answer must be in the form $p + qc = 0$ A1 – accept any correct equivalent answer	

Q5.

Question number	Scheme	Marks
	(a) $(a_2 =)\underline{3k+5}$ [must be seen in part (a) or labelled $a_2 =$]	B1 (1)
	(b) $(a_3 =)3(3k+5)+5$	M1
	$= \underline{9k+20}$ (*)	A1cso (2)
	(c)(i) $a_4 = 3(9k+20)+5 \quad (= 27k+65)$	M1
	$\sum_{r=1}^{4} a_r = k + (3k+5) + (9k+20) + (27k+65)$	M1
	(ii) $= 40k + 90$	A1
	$= \underline{10(4k+9)}$ (or explain why divisible by 10)	A1ft (4) 7
(b)	M1 for attempting to find a_3, follow through their $a_2 \neq k$. A1cso for simplifying to printed result with no incorrect working seen.	
(c)	1st M1 for attempting to find a_4. Can allow a slip here e.g. $3(9k+20)$ [i.e. forgot +5] 2nd M1 for attempting sum of 4 relevant terms, follow through their (a) and (b). Must have 4 terms starting with k. Use of arithmetic series formulae at this point is M0A0A0 1st A1 for simplifying to $40k+90$ or better 2nd A1ft for taking out a factor of 10 or dividing by 10 or an explanation in words true $\forall k$. Follow through their sum of 4 terms provided that both Ms are scored and their sum <u>is</u> divisible by 10. A comment is <u>not</u> required. e.g. $\frac{40k+90}{10} = 4k+9$ is OK for this final A1.	
S.C.	$\sum_{r=2}^{5} a_r = 120k + 290 = 10(12k+29)$ can have M1M0A0A1ft.	

Q6.

Question Number	Scheme	Marks
Q (a)	$(a_2 =)2k-7$	B1 (1)
(b)	$(a_3 =)2(2k-7)-7$ or $4k-14-7, = 4k-21$ (*)	M1, A1cso (2)
(c)	$(a_4 =)2(4k-21)-7 \quad (=8k-49)$	M1
	$\sum_{r=1}^{4} a_r = k + "(2k-7)" + (4k-21) + "(8k-49)"$	M1
	$k+(2k-7)+(4k-21)+(8k-49) = 15k-77 = 43 \qquad k=8$	M1 A1 (4) [7]
(b)	M1 must see 2(their a_2) - 7 or $2(2k-7)-7$ or $4k-14-7$. Their a_2 must be a function of *k*. A1cso must see the $2(2k-7)-7$ or $4k-14-7$ expression and the $4k$ - 21 with no incorrect working	
(c)	1st M1 for an attempt to find a_4 using the given rule. Can be awarded for $8k$ - 49 seen. **Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks.** 2nd M1 for attempting the sum of the 1st 4 terms. Must have "+" not just , or clear attempt to sum. Follow through their a_2 and a_4 provided they are linear functions of *k*. Must lead to linear expression in *k*. Condone use of their linear $a_3 \neq 4k-21$ here too. 3rd M1 for forming a linear equation in *k* using their sum and the 43 and attempt to solve for *k* as far as $pk = q$ A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0 <u>Answer Only</u> (e.g. trial improvement) Accept $k = 8$ <u>only if</u> 8 + 9 + 11 + 15 = 43 is seen as well <u>Sum</u> $\underline{a_2+a_3+a_4+a_5}$ or $\underline{a_2+a_3+a_4}$ Allow: M1 if $8k$ - 49 is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0	

Q7.

Question Number	Scheme	Marks
(a)	$a_2 = \left(\sqrt{4+3}\right) = \sqrt{7}$	B1
	$a_3 = \sqrt{\text{"their 7"}+3} = \sqrt{10}$	B1ft (2)
(b)	$a_4 = \sqrt{10+3}\left(=\sqrt{13}\right)$	M1
	$a_5 = \sqrt{13+3} = 4$ *	A1 cso (2)
		4
	Notes	
(a)	1st B1 for $\sqrt{7}$ only 2nd B1ft follow through their "7" in correct formula provided they have $\sqrt{n}$,where n is an integer.	
(b)	M1 for an attempt to find a_4. Should see $\sqrt{\text{"their"}(a_3)^2+3}$. Must see evidence for M1. $a_4 = \sqrt{13}$ provided this follows from their a_3 working or answer is sufficient A1cso for a correct solution (M1 explicit) must include the = 4. Ending at $\sqrt{16}$ only is A0 and ending with $\pm$ 4 is A0. Ignore any incorrect statements that are not used e.g. common difference = $\sqrt{3}$ <u>Listing</u>: A <u>full</u> list: $2\left(=\sqrt{4}\right)$, $\sqrt{7}$, $\sqrt{10}$, $\sqrt{13}$, $\sqrt{16}=4$ is fine for M1A1	
ALT	<u>Formula</u>: Some may state (or use) $a_n = \sqrt{3n+1}$ leading to $a_5 = \sqrt{3\times 5+1} = 4$. This will get marks in (a) [if correct values are seen] and can score the M1 in (b) if $a_n = \sqrt{3n+1}$ or $a_4 = \sqrt{13}$ are seen.	
$\pm\sqrt{\ }$	If $\pm\sqrt{\ }$ appear any where ignore in part (a) and withhold the final A mark only	

Q8.

Question number	Scheme	Marks
	(a) $1(p+1)$ or $p+1$	B1 (1)
	(b) $((a))(p+(a))$ [(a) must be a function of p]. $[(p+1)(p+p+1)]$	M1
	$=1+3p+2p^2$ (*)	A1cso (2)
	(c) $1+3p+2p^2=1$	M1
	$p(2p+3)=0$ $p=...$	M1
	$p=-\frac{3}{2}$ (ignore $p=0$, if seen, even if 'chosen' as the answer)	A1 (3)
	(d) Noting that even terms are the same.	M1
	This M mark can be implied by listing at least 4 terms, e.g. 1, $-\frac{1}{2}$, 1, $-\frac{1}{2}$,...	
	$x_{2008}=-\frac{1}{2}$	A1 (2)
		8
	(b) M: Valid attempt to use the given recurrence relation to find x_3. Missing brackets, e.g. $p+1(p+p+1)$ Condone for the M1, then if all terms in the expansion are correct, with working fully shown, M1 A1 is still allowed. Beware 'working back from the answer', e.g. $1+3p+2p^2=(1+p)(1+2p)$ scores no marks unless the recurrence relation is justified. (c) 2nd M: Attempt to solve a quadratic equation in p (e.g. quadratic formula or completing the square). The equation must be based on $x_3=1$. The attempt must lead to a non-zero solution, so just stating the zero solution $p=0$ is M0. A: The A mark is dependent on both M marks. (d) M: Can be implied by a correct answer for their p (answer is $p+1$), and can also be implied if the working is 'obscure'). Trivialising, e.g. $p=0$, so every term = 1, is M0. If the additional answer $x_{2008}=1$ (from $p=0$) is seen, ignore this (isw).	

Q9.

Question	Scheme	Marks	AOs
(a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n+2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2),\ a_2 = 2k,\ a_3 = k+1,\ a_4 = \frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1} = 2 \Rightarrow k^2+3k = 2k+2 \Rightarrow k^2+k-2=0$ *	A1*	2.1
		(3)	
(b)	States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26\times(2+-4+-1)+2+-4$	M1	3.1a
	$=-80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

Q10.

Question Number	Scheme		Marks
(a)	$(a_2 =)2k$	$2k$ only	B1
	$(a_3 =)\frac{k("2k"+1)}{"2k"}$	For substituting their a_2 into $a_3 = \frac{k(a_2+1)}{a_2}$ to find a_3 in terms of just k	M1
	$(a_3 =)\frac{2k+1}{2}$	$(a_3 =)\frac{2k+1}{2}$ or exact simplified equivalent such as $(a_3 =)k+\frac{1}{2}$ or $\frac{1}{2}(2k+1)$ but not $k+\frac{k}{2k}$ **Must be seen in (a)** but isw once a correct simplified answer is seen.	A1
			(3)
	Note that there are no marks in (b) for using an AP (or GP) sum formula unless their terms do form an AP (or GP).		
(b)	$\sum_{r=1}^{3} a_r = 10 \Rightarrow 1+"2k"+"\frac{2k+1}{2}"=10$	Writes 1 + their a_2 + their a_3 = 10. E.g. $1+2k+\frac{2k^2+k}{2k}=10$. Must be a correct follow through equation in terms of k only.	M1
	$\Rightarrow 2+4k+2k+1=20 \Rightarrow k=...$ or e.g. $\Rightarrow 6k^2-17k=0 \Rightarrow k=...$	Solves their equation in k which has come from the sum of 3 terms = 10, and reaches $k=...$ Condone poor algebra but if a quadratic is obtained then the usual rules apply for solving – see General Principles. (Note that it does not need to be a 3-term quadratic in this case)	M1
	$(k =)\frac{17}{6}$	$k=\frac{17}{6}$ or exact equivalent e.g. $2\frac{5}{6}$ Do **not** allow $k=\frac{8.5}{3}$ or $k=\frac{17/2}{3}$ Ignore any reference to $k=0$. Allow 2.83 recurring as long as the recurring is clearly indicated e.g. a dot over the 3.	A1
			(3)
			(6 marks)

Q11.

Question Number	Scheme		Marks
(a)	$(a_2 =)2k$	$2k$ only	B1
	$(a_3 =)\dfrac{k("2k"+1)}{"2k"}$	For substituting their a_2 into $a_3 = \dfrac{k(a_2+1)}{a_2}$ to find a_3 in terms of just k	M1
	$(a_3 =)\dfrac{2k+1}{2}$	$(a_3 =)\dfrac{2k+1}{2}$ or exact simplified equivalent such as $(a_3 =)k+\dfrac{1}{2}$ or $\dfrac{1}{2}(2k+1)$ but not $k+\dfrac{k}{2k}$ **Must be seen in (a)** but isw once a correct simplified answer is seen.	A1
			(3)
	Note that there are no marks in (b) for using an AP (or GP) sum formula unless their terms do form an AP (or GP).		
(b)	$\sum_{r=1}^{3} a_r = 10 \Rightarrow 1+"2k"+"\dfrac{2k+1}{2}"=10$	Writes 1 + their a_2 + their a_3 = 10. E.g. $1+2k+\dfrac{2k^2+k}{2k}=10$. Must be a correct follow through equation in terms of k only.	M1
	$\Rightarrow 2+4k+2k+1=20 \Rightarrow k=...$ or e.g. $\Rightarrow 6k^2-17k=0 \Rightarrow k=...$	Solves their equation in k which has come from the sum of 3 terms = 10, and reaches $k=...$ Condone poor algebra but if a quadratic is obtained then the usual rules apply for solving – see General Principles. (Note that it does not need to be a 3-term quadratic in this case)	M1
	$(k=)\dfrac{17}{6}$	$k=\dfrac{17}{6}$ or exact equivalent e.g. $2\dfrac{5}{6}$ Do **not** allow $k=\dfrac{8.5}{3}$ or $k=\dfrac{17/2}{3}$ Ignore any reference to $k=0$. Allow 2.83 recurring as long as the recurring is clearly indicated e.g. a dot over the 3.	A1
			(3)
			(6 marks)

Q12.

Question Number	Scheme	Marks
(a)	$[x_2 =]\, a-3$	B1 (1)
(b)	$[x_3 =]\ ax_2 - 3$ or $a(a-3)-3$	B1
	$= a(a-3) - {}_{5} \ = a^2 - 3a - 3$ (*)	A1 cso (2)
(c)	$a^2 - 3a - 3 = 7$	
	$a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$	M1
	$(a-5)(a+2) = 0$	M1
	$a = 5$ or -2	A1 (3)
		(6 marks)