Implicit Differentiation (Sheet 2)

Q1.
Find the gradient of the curve with equation

$\ln y = 2x \ln x, \quad x > 0,\ y > 0$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

(Total 7 marks)

Q2.
A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

(Total 9 marks)

Q3.
A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

(Total 9 marks)

Q4.
The curve C has the equation

$$\cos 2x + \cos 3y = 1, \qquad -\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}, \quad 0 \leqslant y \leqslant \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.
(b) Find the value of y at P.

(3)

(c) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)

(Total 9 marks)

Q5.

The curve C has equation

$$x^2 - 3xy - 4y^2 + 64 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total for question = 11 marks)

Q6.

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(a) Find $^{dy}/_{dx}$ in terms of x and y, fully simplifying your answer.

(5)

(b) Find the values of y for which $^{dy}/_{dx} = 0$

(5)

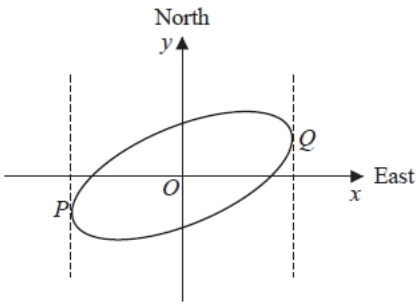
(Total 10 marks)

Q7.

The figure shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\frac{dy}{dx} = \frac{y - x}{3y - x}$

(4)



The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

(Total for question = 10 marks)

Q8.
The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

(Total for question = 9 marks)

Q9.
The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{dy}{dx}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)

(Total 7 marks)

Q10.
The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

(Total for question = 9 marks)

Q11.
A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(7)
(Total 7 marks)