Implicit Differentiation (Sheet 2) Mark Scheme

Q1.

Question Number	Scheme		Marks	
		$\frac{1}{y}\frac{dy}{dx} = \ldots$	B1	
		$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
	At $x = 2$,	$\ln y = 2(2)\ln 2$	M1	
	leading to	$y = 16$ Accept $y = e^{4\ln 2}$	A1	
	At $(2, 16)$	$\frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$	M1	
		$\frac{dy}{dx} = 16(2 + 2\ln 2)$	A1	(7)
				[7]
	Alternative			
		$y = e^{2x\ln x}$	B1	
		$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
		$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$	M1 A1	
	At $x = 2$,	$\frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$	M1	
		$= 16(2 + 2\ln 2)$	A1	(7)

Q2.

Question Number	Scheme		Marks
(a)	$x^3 - 4y^2 = 12xy \quad (\text{ eqn } *)$		
	$x = -8 \Rightarrow \; -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by *completing the square*.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \frac{\cancel{dx}}{\cancel{dx}} \times \right\} \quad 3x^2 - 8y\frac{dy}{dx} = \left(\underline{12y + 12x\frac{dy}{dx}} \right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \ldots$ Correct LHS equation; Correct application of product rule	M1 A1; (B1)
	$\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ *correctly* found.	dM1 A1 A1 cso
			[6]
			9 marks

Q3.

Question Number	Scheme	Marks
(a)	$3x^2 - y^2 + xy = 4$ (eqn *)	
	$\left\{\frac{dx}{dx} \times\right\}$ $\underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	M1 B1 A1
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	M1
	giving $-18x - 3y = 8x - 16y$	
	giving $13y = 26x$	M1
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	A1 cso (6)
(b)	At P & Q, $y = 2x$. Substituting into eqn *	
	gives $3x^2 - (2x)^2 + x(2x) = 4$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	A1
	$y = 2x \Rightarrow y = \pm 4$, hence coordinates are $\underline{(2, 4)}$ and $\underline{(-2, -4)}$	A1 (3)
		(9 marks)

Q4.

Question Number	Scheme	Marks
	(a) $-2\sin 2x - 3\sin 3y\frac{dy}{dx} = 0$	M1 A1
	$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}, \frac{-2\sin 2x}{3\sin 3y}$	A1 (3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$	M1
	$\cos 3y = \frac{1}{2}$	A1
	$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349	A1 (3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$	M1
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$	M1
	Leading to $6x + 9y - 2\pi = 0$	A1 (3) **[9]**

Q5.

Question Number	Scheme		Marks
	$x^2 - 3xy - 4y^2 + 64 = 0$		
(a)	$\left\{\frac{dy}{dx} = \right\}$ $2x - \left(3y + 3x\frac{dy}{dx}\right) - 8y\frac{dy}{dx} = 0$		M1A1 M1
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ o.e.		A1 cso
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow \right\}$ $2x - 3y = 0$		M1
	$y = \frac{2}{3}x$	$x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$	$\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$	$\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}$ $x = \frac{24}{5}$ or $-\frac{24}{5}$	$\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\}$ $y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 cso
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$	When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ or $x = \frac{24}{5}$, $y = \frac{16}{5}$ and $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ cso		ddM1 A1
			[6] 11

Q6.

Question Number	Scheme		Marks
	$x^2 + y^2 + 10x + 2y - 4xy = 10$		
(a)	$\left\{\frac{dy}{dx} \times\right\}$ $\underline{2x + 2y\frac{dy}{dx}} + 10 + \underline{2\frac{dy}{dx}} - \left(\underline{\underline{4y + 4x\frac{dy}{dx}}}\right) = \underline{0}$	**See notes**	M1 A1 M1
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{dy}{dx} = 0$	**Dependent on the first M1 mark.**	dM1
	$\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$		
	Simplifying gives $\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1}$ $\left\{= \frac{-x - 5 + 2y}{-2x + y + 1}\right\}$		A1 cso oe
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $x + 5 - 2y = 0$		M1
	So $x = 2y - 5$,		
	$(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$		M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$		
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35 \{= 0\}$ **see notes**	A1 oe
	$(3y - 7)(y - 5) = 0$ and $y = ...$	Method mark for solving a quadratic equation.	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\} \frac{7}{3}, 5$	A1 cao
			[5]
	<u>Alternative method for part (b)</u>		
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $x + 5 - 2y = 0$		M1
	So $y = \frac{x + 5}{2}$,		
	$x^2 + \left(\frac{x + 5}{2}\right)^2 + 10x + 2\left(\frac{x + 5}{2}\right) - 4x\left(\frac{x + 5}{2}\right) = 10$		M1
	$x^2 + \frac{x^2 + 10x + 25}{4} + 10x + x + 5 - 2x^2 - 10x = 10$		
	$4x^2 + x^2 + 10x + 25 + 40x + 4x + 20 - 8x^2 - 40x = 40$		
	gives $-3x^2 + 14x + 5 = 0$ or $3x^2 - 14x - 5 = 0$	$3x^2 - 14x - 5 \{= 0\}$ **see notes**	A1 oe
	$(3x + 1)(x - 5) = 0,\ x = ...$		
	$y = \frac{-\frac{1}{3} + 5}{2}, \frac{5 + 5}{2}$	**Solves a quadratic and finds at least one value for y.**	ddM1
	$y = \frac{7}{3}, 5$	$\{y =\} \frac{7}{3}, 5$	A1 cao
			[5]
			10

Q7.

Question	Scheme	Marks	AOs
(a)	Either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx} + 2y$	M1	2.1
	$2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx} = 0$	A1	1.1b
	$(6y - 2x)\frac{dy}{dx} = 2y - 2x$	M1	2.1
	$\frac{dy}{dx} = \frac{2y-2x}{6y-2x} = \frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{dy}{dx} \to \infty \Rightarrow\right)$ Deduces that $3y - x = 0$	M1	2.2a
	Solves $y = \frac{1}{3}x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously	M1	3.1a
	$\Rightarrow x = (\pm)5\sqrt{3}$ OR $\Rightarrow y = (\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y = \frac{1}{3}x \Rightarrow x = ..$ AND $y = ..$	dM1	1.1b
	$P = \left(-5\sqrt{3}, -\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Q8.

Question	Scheme	Marks	AOs
(a)	$\frac{d}{dx}(3y^2) = 6y\frac{dy}{dx}$ or $\frac{d}{dx}(qxy) = qx\frac{dy}{dx} + qy$	M1	2.1
	$3px^2 + qx\frac{dy}{dx} + qy + 6y\frac{dy}{dx} = 0$	A1	1.1b
	$(qx+6y)\frac{dy}{dx} = -3px^2 - qy \Rightarrow \frac{dy}{dx} = ...$	dM1	2.1
	$\frac{dy}{dx} = \frac{-3px^2 - qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3 + q(-1)(-4) + 3(-4)^2 = 26$	M1	1.1b
	$19x + 26y + 123 = 0 \Rightarrow m = -\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2 - q(-4)}{q(-1)+6(-4)} = \frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2+q(-4)} = -\frac{19}{26}$	M1	3.1a
	$p - 4q = 22,\ 57p - 102q = 624 \Rightarrow p = ...,q = ...$	dM1	1.1b
	$p = 2,\ q = -5$	A1	1.1b
		(5)	
			(9 marks)

Q9.

Question Number	Scheme		Marks
	$3^{x-1} + xy - y^2 + 5 = 0$		
		$3^{x-1} \rightarrow 3^{x-1}\ln 3$	B1 oe
	$\left\{\frac{dy}{dx} \times\right\}$ (ignore) $\quad 3^{x-1}\ln 3 + \left(y + x\frac{dy}{dx}\right) - 2y\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$xy \rightarrow +y + x\frac{dy}{dx}$	B1
		$\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	A1
	$\{(1,3) \Rightarrow\}\ 3^{(1-1)}\ln 3 + 3 + (1)\frac{dy}{dx} - 2(3)\frac{dy}{dx} = 0$	Substitutes $x = 1$, $y = 3$ into their differentiated equation or expression.	dM1*
	$\ln 3 + 3 + \frac{dy}{dx} - 6\frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5\frac{dy}{dx}$		
	$\frac{dy}{dx} = \frac{3 + \ln 3}{5}$		dM1*
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 cso
			[7] 7

Notes for Question

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1} \rightarrow 3^{x-1}\ln 3$ or $3^{x-1} = \frac{1}{3}(3^x) \rightarrow \frac{1}{3}(3^x)\ln 3$

or $3^{x-1} = e^{(x-1)\ln 3} \rightarrow \ln 3\, e^{(x-1)\ln 3}$ or $3^{x-1} = \frac{1}{3}(3^x) = \frac{1}{3}e^{x\ln 3} \rightarrow \frac{1}{3}(\ln 3)e^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).

B1: $xy \rightarrow +y + x\frac{dy}{dx}$

1st A1: $\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "= 0" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3 + y + x\frac{dy}{dx} - 2y\frac{dy}{dx}$ leading to $3^{x-1}\ln 3 + y = 2y\frac{dy}{dx} - x\frac{dy}{dx}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x = 1$, $y = 3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{dy}{dx}$ and rearranges to make $\frac{dy}{dx}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{dy}{dx} = \frac{y + 3^{x-1}\ln 3}{2y - x}$ before substituting in $x = 1$ and $y = 3$

2nd A1: cso. Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$, $\left(= \frac{1}{\lambda}\ln\left(\mu e^3\right),\ \lambda = 5 \text{ and } \mu = 3\right)$

Note: $3 = \ln e^3$ needs to be seen in their proof.

Q10.

Question Number	Scheme	Notes	Marks
	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) Way 1	$\left\{\frac{dy}{dx} \times\right\} 8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y \ln 2\frac{dy}{dx} = 0$		M1 A1 M1 B1
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4 \ln 2\frac{dy}{dx} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 cso
	NOTE: You can recover work for part (a) in part (b)		[6]
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N ($\neq m_T$), either $y - 4 = m_N(x - -2)$ **and sets** $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}\right.$, so $\left.y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$		
	y (or c) $= \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ or $\frac{13}{2} - \ln 2$ or $-\ln 2 + \frac{13}{2}$	A1 cso isw
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		[3]
			9

Question Number	Scheme	Notes	Marks
(a) Way 2	$\left\{\frac{dx}{dy} \times\right\} 8x\frac{dx}{dy} - 3y^2 - 4y\frac{dx}{dy} - 4x + 2^y \ln 2 = 0$		M1 A1 M1 B1
	$8(-2)\frac{dx}{dy} - 3(4)^2 - 4(4)\frac{dx}{dy} - 4(-2) + 2^4 \ln 2 = 0$	**dependent on the first M mark**	dM1
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 cso
	Note: You must be clear that Way 2 is being applied before you use this scheme		[6]

Question Notes

(a)	**Note**	**For the first four marks**
		Writing down *from no working* • $\frac{dy}{dx} = \frac{4y-8x}{-3y^2-4x+2^y\ln 2}$ or $\frac{8x-4y}{3y^2+4x-2^y\ln 2}$ scores M1A1M1B1 • $\frac{dy}{dx} = \frac{8x-4y}{-3y^2-4x+2^y\ln 2}$ or $\frac{4y-8x}{3y^2+4x-2^y\ln 2}$ scores M1A0M1B1
		Writing $8x\,dx - 3y^2\,dy - 4y\,dx - 4x\,dy + 2^y\ln 2\,dy = 0$ scores M1A1M1B1

	Question Notes Continued	
(a)	1st M1	Differentiates implicitly to include *either* $\pm 4x\frac{dy}{dx}$ *or* $-y^3 \to \pm\lambda y^2\frac{dy}{dx}$ *or* $2^y \to \pm\mu 2^y\frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx}=\right)$). λ, μ are constants which can be 1
	1st A1	**Both** $4x^2 - y^3 \to 8x - 3y^2\frac{dy}{dx}$ **and** $=0 \to =0$
	Note	e.g. $8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y\ln 2\frac{dy}{dx} \to -3y^2\frac{dy}{dx} - 4x\frac{dy}{dx} + 2^y\ln 2\frac{dy}{dx} = 4y - 8x$ or e.g. $-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} \to -48\frac{dy}{dx} + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 32$ will get 1st A1 (implied) as the "$=0$" can be implied by the rearrangement of their equation.
	2nd M1	$-4xy \to -4y - 4x\frac{dy}{dx}$ or $4y - 4x\frac{dy}{dx}$ or $-4y + 4x\frac{dy}{dx}$ or $4y + 4x\frac{dy}{dx}$
	B1	$2^y \to 2^y\ln 2\frac{dy}{dx}$ **or** $2^y \to e^{y\ln 2}\ln 2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0
	3rd dM1	**dependent on the first M mark** For substituting $x=-2$ and $y=4$ into an equation involving $\frac{dy}{dx}$
	Note	M1 can be gained by seeing at least one example of substituting $x=-2$ and at least one example of substituting $y=4$ **unless** it is clear that they are instead applying $x=4$ and $y=-2$ Otherwise, you will NEED to check (with your calculator) that $x=-2$, $y=4$ that has been substituted into their equation involving $\frac{dy}{dx}$
	Note	**A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**isw:** You can, however, ignore subsequent working following on from correct solution.
(b)	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 1st M1 and 2nd M1 for** $\frac{y-4}{2} = \frac{-1}{\text{their } m_T \text{ evaluated at } x=-2 \text{ and } y=4}$
	Note	**A1:** Allow the alternative answer $\{y=\}\ \ln\left(\frac{1}{2}\right) + \frac{13}{2\ln 2}(\ln 2)$ which is in the form $p + q\ln 2$

(a) Way 2	1st M1	Differentiates implicitly to include *either* $\pm 4y\frac{dx}{dy}$ *or* $4x^2 \to \pm\lambda x\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy}=\right)$). λ is a constant which can be 1
	1st A1	**Both** $4x^2 - y^3 \to 8x\frac{dx}{dy} - 3y^2$ **and** $=0 \to =0$
	2nd M1	$-4xy \to -4y\frac{dx}{dy} - 4x$ or $4y\frac{dx}{dy} - 4x$ or $-4y\frac{dx}{dy} + 4x$ or $4y\frac{dx}{dy} + 4x$
	B1	$2^y \to 2^y\ln 2$
	3rd dM1	**dependent on the first M mark** For substituting $x=-2$ and $y=4$ into an equation involving $\frac{dx}{dy}$

Q11.

Question Number	Scheme		Marks	
	$\frac{d}{dx}\left(2^x\right) = \ln 2 . 2^x$		B1	
	$\ln 2 . 2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$		M1 A1= A1	
	Substituting $(3, 2)$			
	$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$		M1	
	$\frac{dy}{dx} = 4\ln 2 - 2$	Accept exact equivalents	M1 A1	(7)
				[7]