Question 1 (Jun 2007, Q1)

Worked Solution

A geometric progression $u_1, u_2, u_3, \ldots$ with $u_1 = 15$ and $u_{n+1} = 0.8\,u_n$.

(i) Write down u_2, u_3 and u_4

The common ratio is $r = 0.8$:

$$u_2 = 15 \times 0.8 = 12$$

$$u_3 = 12 \times 0.8 = 9.6$$

$$u_4 = 9.6 \times 0.8 = 7.68$$

$u_2 = 12, \quad u_3 = 9.6, \quad u_4 = 7.68$

(ii) Find $\sum_{n=1}^{20} u_n$

This is a geometric series with $a = 15$, $r = 0.8$, $n = 20$:

$$S_{20} = \frac{15(1 - 0.8^{20})}{1 - 0.8} = \frac{15(1 - 0.8^{20})}{0.2}$$

$$0.8^{20} = 0.011529\ldots$$

$$S_{20} = \frac{15 \times 0.988471}{0.2} = \frac{14.8271}{0.2} = 74.135\ldots$$

$$\sum_{n=1}^{20} u_n = 74.1 \text{ (to 3 s.f.)}$$

Question 2 (Jan 2008, Q8)

Worked Solution

First term $a = 10$, common ratio $r = 0.8$.

(i) Find the fourth term

$$u_4 = 10 \times 0.8^3 = 10 \times 0.512$$

$u_4 = 5.12$

(ii) Sum of first 20 terms (to 3 s.f.)

$$S_{20} = \frac{10(1 - 0.8^{20})}{1 - 0.8} = \frac{10(1 - 0.011529)}{0.2} = \frac{10 \times 0.988471}{0.2} = \frac{9.88471}{0.2}$$

$S_{20} = 49.4$ (to 3 s.f.)

(iii) Show $S_\infty - S_N < 0.01$ can be written as $0.8^N < 0.0002$, and find smallest N

Sum to infinity:

$$S_\infty = \frac{10}{1 - 0.8} = 50$$

$$S_N = \frac{10(1 - 0.8^N)}{0.2} = 50(1 - 0.8^N)$$

$$S_\infty - S_N = 50 - 50(1 - 0.8^N) = 50 \times 0.8^N$$

So $S_\infty - S_N < 0.01$ becomes:

$$50 \times 0.8^N < 0.01 \implies 0.8^N < 0.0002 \checkmark$$

Taking logarithms (note $\log 0.8 < 0$, so inequality reverses):

$$N \log 0.8 < \log 0.0002 \implies N > \frac{\log 0.0002}{\log 0.8} = \frac{-3.69897}{-0.09691} = 38.168\ldots$$

Smallest $N = 39$

Question 3 (Jan 2011, Q5)

Worked Solution

Sum to infinity is four times the first term.

(i) Show the common ratio is $\frac{3}{4}$

$$S_\infty = 4a \implies \frac{a}{1-r} = 4a$$

Dividing by a (since $a \neq 0$):

$$\frac{1}{1-r} = 4 \implies 1 = 4(1-r) = 4 - 4r \implies 4r = 3$$

$r = \dfrac{3}{4}$ ✓

(ii) Given third term is 9, find the first term

Third term $= ar^2 = 9$:

$$a\left(\frac{3}{4}\right)^2 = 9 \implies a \times \frac{9}{16} = 9 \implies a = 16$$

$a = 16$

(iii) Sum of first 20 terms

$$S_{20} = \frac{16\left(1-\left(\frac{3}{4}\right)^{20}\right)}{1-\frac{3}{4}} = \frac{16\left(1-\left(\frac{3}{4}\right)^{20}\right)}{\frac{1}{4}} = 64\left(1-\left(\frac{3}{4}\right)^{20}\right)$$

$$\left(\frac{3}{4}\right)^{20} = 0.003171\ldots$$

$$S_{20} = 64 \times 0.996829 = 63.797\ldots$$

$S_{20} = 63.8$ (to 3 s.f.)

Question 4 (Jun 2013, Q6)

Worked Solution

First experiment uses 6 g, second uses 7.8 g.

(i) Arithmetic progression: total for first 30 experiments

Common difference $d = 7.8 - 6 = 1.8$, $a = 6$, $n = 30$:

$$S_{30} = \frac{30}{2}\big(2 \times 6 + 29 \times 1.8\big) = 15\,(12 + 52.2) = 15 \times 64.2$$

$S_{30} = 963\,\text{g}$

(ii) Geometric progression: show $1.3^N \leq 91$, and find N

Common ratio $r = \frac{7.8}{6} = 1.3$. Total after N experiments:

$$S_N = \frac{6(1.3^N - 1)}{1.3 - 1} = \frac{6(1.3^N - 1)}{0.3} = 20(1.3^N - 1) \leq 1800$$

$$1.3^N - 1 \leq 90 \implies 1.3^N \leq 91 \checkmark$$

Taking logarithms:

$$N \log 1.3 \leq \log 91$$

$$N \leq \frac{\log 91}{\log 1.3} = \frac{1.95904}{0.11394} = 17.194\ldots$$

$N = 17$

End of Worked Solutions