Geometric Sequences Exam Questions Sheet 2

Q1.
A geometric series is $a + ar + ar^2 + ...$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term,

(2)

(d) the sum to infinity.

(3)

(Total 11 marks)

Q2.
A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

(Total 9 marks)

Q3.
The first three terms of a geometric series are

18, 12 and p

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series,

(1)

(b) the value of p,

(1)

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

(Total 4 marks)

Q4.

A geometric series has first term a, where $a \neq 0$, and common ratio r.
The sum to infinity of this series is 6 times the first term of the series.

(a) Show that $r = \frac{5}{6}$

(2)

Given that the fourth term of this series is 62.5

(b) find the value of a,

(2)

(c) find the difference between the sum to infinity and the sum of the first 30 terms, giving your answer to 3 significant figures.

(4)

(Total 8 marks)

Q5.
The first three terms of a geometric sequence are

$$7k - 5,\ 5k - 7,\ 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \frac{9}{11}$

(2)

For this value of k,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

(Total for question = 12 marks)

Q6.
The second and fifth terms of a geometric series are 750 and −6 respectively.

Find

(a) the common ratio of the series,

(3)

(b) the first term of the series,

(2)

(c) the sum to infinity of the series.

(2)

(Total 7 marks)

Q7.

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum to infinity,

(2)

(d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

(Total 10 marks)

Q8.

The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

(Total 10 marks)

Q9.

A car was purchased for £18 000 on 1st January.

On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.

(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(b) Find the value of n.

(3)

An insurance company has a scheme to cover the maintenance of the car.

The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.

(2)

(d) Find the total cost of the insurance scheme for the first 15 years.

(3)

(Total 9 marks)

Q10.

A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

(Total 9 marks)

Q11.

The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$

(4)

(b) Hence show that $p = 5$

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

(3)

(Total 11 marks)