Geometric Sequences Exam Questions Sheet 2 Mark Scheme

Q1.

Question	Scheme		Marks
(a)	$(S_n =)\ a + ar + (ar^2) + \ldots \quad + ar^{n-1}$ **and** $rS_n = ar + ar^2 + (ar^3)\ldots \quad + ar^n$		M1
	$S_n - rS_n = a - ar^n$		M1
	$S_n(1-r) = a(1-r^n)$		dM1
	And so result $S_n = \dfrac{a(1-r^n)}{(1-r)}$ *		A1 (4)
(b)	Divides one term by other (either way) to give $r^2 = \ldots$ then square roots to give $r =$	**Or:** (*Method 2)* Finds geometric mean i.e 3.24 and divides one term by 3.24 or 3.24 by one term	M1
	$r^2 = \dfrac{1.944}{5.4}$, $\quad r = 0.6$ (ignore – 0.6)	$r = 0.6$ (ignore – 0.6)	A1 (2)
(c)	Uses $\quad 5.4 \div r^2$ or $1.944 \div r^4$, to give $a =$ $a = 15$		M1, A1ft (2)
(d)	Uses $\quad S = \dfrac{15}{1-0.6}$, to obtain 37.5		M1A1 ,A1 (3)
			11 marks
Notes Special Case	**(a) M1:** Lists **both** of these sums ($S_n =$) may be omitted, rS_n (or rS) must be stated 1[st] two terms must be correct in each series. Last term must be ar^{n-1} or ar^n in first series and the corresponding ar^n or ar^{n+1} in second series. Must be n and not a number. Reference made to other terms e.g. space or dots to indicate missing terms **M1:** Subtracts series for rS from series for S (or other way round) to give RHS $= \pm(a - ar^n)$. This may have been obtained by following a pattern. **If wrong power stated on line 1 M0 here**. (Ignore LHS)M0M0M0A0 **dM1**: Factorises both sides correctly– must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) **A1**: completes the proof with **no errors** seen No errors seen: **First line absolutely correct**, omission of second line, third and fourth lines correct: M1M0M1A1 See next sheet of common errors. Refer any attempts involving **sigma notation**, or any **proofs by induction** to team leader. Also attempts which **begin with the answer and work backwards**. **(b) M1:** Deduces r^2 by dividing either term by other and attempts square root **A1:** any correct equivalent for r e.g. 3/5 Answer only is 2/2 (*Method 2*) Those who find fourth term must use $\sqrt{ab}$ and **not** $\frac{1}{2}(a+b)$ then must use it in a division with given term to obtain $r =$ **(c) M1:** May be done in two steps or more e.g. $5.4 \div r$ then divided by r again **A1ft:** follow through their value of r. Just $a = 15$ **with no wrong working** implies M1A1 (d) **M1**: States sum to infinity formula with values of a and r found earlier, provided $\lvert r \rvert < 1$ **A1** : uses 15 and 0.6 (or 3/5) (This is **not** a ft mark) **A1**: 37.5 or exact equivalent		
Common errors	(i) Fraction inverted in (b) $r^2 = \dfrac{5.4}{1.944}$ and $r = 1\frac{2}{3}$, then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. 3/7 (ii) Uses $r = 0.36$: (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. 3/7 (iii) Uses $ar^2 = 5.4$, $\ ar^5 = 1.944$ Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e.3/7		

Q2.

Question Number	Scheme		Marks
(a)	$120000\times(1.05)^3 = 138915$*	Or $120000\times1.05\times1.05\times1.05 = 138915$ Or 120000, 126000, 132300, 138915 Or $a = 120000$ and $a\times(1.05)^3 = 138915$	B1
			(1)
(b)	$120000\times(1.05)^{n-1} > 200000$	Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$\log 1.05^{n-1} > \log\left(\frac{5}{3}\right)$	Takes logs correctly Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$(n-1>)\frac{\log\left(\frac{5}{3}\right)}{\log 1.05}$ or equivalent e.g $(n>)\frac{\log\left(\frac{7}{4}\right)}{\log 1.05}$	Allow n or $n-1$ and ">", "<", or "=" etc. Allow $1.\dot{6}$ or awrt 1.67 for 5/3.	A1
	2024	M1: Identifies a calendar year using their value of n or n - 1 A1: 2024	M1A1
			(5)
(c)	$\frac{a(1-r^n)}{1-r} = \frac{120000(1-1.05^{11})}{1-1.05}$	M1: Correct sum formula with $n = 10, 11$ or 12 A1: Correct numerical expression with $n = 11$	M1 A1
	1704814	Cao (Allow 1704814.00)	A1
			(3)
			[9]
	Listing or trial/improvement in (b)		
	$U_{10} = 186\ 159.39$, $U_{11} = 195\ 467.36$, $U_{12} = 205\ 240.72$		
	Attempt to find at least the 10th or 11th or 12th terms correctly using a common ratio of 1.05 (all the terms need **not** be listed)		M1
	Forms the geometric progression correctly to reach a term > 200 000		M1
	Obtains an "11th" term of awrt 195 500 and a "12th" term of awrt 205 200		A1
	Uses their number of terms to identify a calendar year		M1
	2024		A1
			(5)

Q3.

Question Number	Scheme	Marks
(a)	$\{r=\}\ \frac{2}{3}$	B1
		(1)
(b)	$\{p=\}\ 8$	B1 cao
		(1)
(c)	$\{S_{15}=\}\ \frac{18\left(1-\left(\frac{2}{3}\right)^{15}\right)}{1-\frac{2}{3}}$	M1
	$\{S_{15}=53.87668...\} \Rightarrow S_{15}=\text{awrt } 53.877$	A1
		(2)
		[4]
	Notes for Question	
(a)	B1: Accept $\frac{12}{18}$, $0.\dot{6}$ or 0.6 recurring, or even 0.667 (3sf) but not 0.6 or 0.67	
(b)	B1: accept 8 only	
(c)	M1: Applies this formula $S_{15}=\frac{18\left(1-(\text{their } r)^{15}\right)}{1-(\text{their } r)}$, can be implied by their answer. For this mark they may use any value for r except $r=1$ or $r=0$ (even 3/2 or -6 may be used) A1: Answers which round to 53.877	
Alternative method for (c)	M1: (Adding terms is an unlikely method for this question) Need to see 15 terms listed as 18+12+……0.06165877 or can be implied by correct answer A1: awrt 53.877 **Answer only** : 53.9 is M0A0 with no working, but 53.877 with no working is M1A1	

Q4.

Question Number	Scheme		Marks
(a)	$S_\infty = 6a$		
	$\frac{a}{1-r} = 6a$	Either $\frac{a}{1-r} = 6a$ or $\frac{6a}{1-r} = a$ or $\frac{6}{1-r} = 1$	M1
	$\{\Rightarrow 1 = 6(1-r) \Rightarrow\}\ r = \frac{5}{6}$*	cso	A1*
	Allow verification e.g. $\frac{a}{1-r} = 6a \Rightarrow \frac{a}{1-\frac{5}{6}} = 6a \Rightarrow \frac{a}{\frac{1}{6}} = 6a \Rightarrow 6a = 6a$		
			[2]
(b)	$\{T_4 = ar^3 = 62.5 \Rightarrow\}\ a\left(\frac{5}{6}\right)^3 = 62.5$	$a\left(\frac{5}{6}\right)^3 = 62.5$ (Correct statement using the 4th term. Do not accept $a\left(\frac{5}{6}\right)^4 = 62.5$)	M1
	$\Rightarrow a = 108$	108	A1
			[2]
(c)	$S_\infty = 6(\text{their } a)$ or $\frac{\text{their } a}{1-\frac{5}{6}}\{= 648\}$	Correct method to find S_∞	M1
	$\{S_{30} =\}\ \frac{108\left(1-\left(\frac{5}{6}\right)^{30}\right)}{1-\frac{5}{6}}\ \{= 645.2701573...\}$	M1: $S_{30} = \frac{(\text{their } a)\left(1-\left(\frac{5}{6}\right)^{30}\right)}{1-\left(\frac{5}{6}\right)}$ **(Condone invisible brackets around 5/6)** A1ft: Correct follow through expression (follow through their a). **Do not condone invisible brackets around 5/6 unless <u>their</u> evaluation or final answer implies they were intended.**	M1 A1ft
	$\{S_\infty - S_{30}\} = 2.72984...$	awrt 2.73	A1
			[4]
			Total 8
(c)	**<u>Alternative:</u>** Difference $= \frac{ar^{30}}{1-r} = \frac{108\left(\frac{5}{6}\right)^{30}}{1-\frac{5}{6}} = 2.72984...$ M1M1: For an attempt to apply $\frac{ar^{30}}{1-r}$. A1ft: $\frac{(\text{their } a)\times r^{30}}{1-r}$ with their ft a. A1: awrt 2.73		

Q5.

Question Number	Scheme	Marks
(a)	$a = 7k-5,\ ar = 5k-7$ and $ar^2 = 2k+10$	B1
	(So $r=$) $\dfrac{5k-7}{7k-5} = \dfrac{2k+10}{5k-7}$ or $(7k-5)(2k+10) = (5k-7)^2$ or equivalent	M1
	See $(5k-7)^2 = 25k^2-70k+49$	M1
	$14k^2+60k-50 = 25k^2-70k+49 \rightarrow\ 11k^2-130k+99=0$ *	A1cso * (4)
(b)	$(k-11)(11k-9)$ so $k=$	M1
	$k = 9/11$ only* (after rejecting 11) **N.B. Special case $k = 9/11$ can be verified in (b) (1 mark only)** $11\times\left(\dfrac{9}{11}\right)^2 - 130\times\left(\dfrac{9}{11}\right)+99 = \dfrac{81}{11} - \dfrac{1170}{11} + \dfrac{1089}{11} = 0$ M1A0	A1* (2)
(c)	$a = \dfrac{8}{11}$	B1
	$\dfrac{5\times\frac{9}{11}-7}{7\times\frac{9}{11}-5}$ *or* $\dfrac{2\times\frac{9}{11}+10}{5\times\frac{9}{11}-7}$ so $r=-4$	B1
	(i) Fourth term $= ar^3 = -\dfrac{512}{11}$	M1A1
	(ii) $S_{10} = \dfrac{a(1-r^{10})}{(1-r)} = \dfrac{\frac{8}{11}(1-(-4)^{10})}{(1-(-4))} = -152520$	M1A1 (6) [12]

Q6.

Question Number	Scheme	Marks
(a)	$ar = 750$ and $ar^4 = -6$ (could be implied from later working in either (a) or (b)).	B1
	$r^3 = \frac{-6}{750}$	M1
	$r = -\frac{1}{5}$ Correct answer from no working, except for special case below gains all three marks.	A1
		(3)
(b)	$a(-0.2) = 750$	M1
	$a\left\{= \frac{750}{-0.2}\right\} = -3750$	A1 ft
		(2)
(c)	Applies $\frac{a}{1-r}$ correctly using both their a and their $\|r\| < 1$. Eg. $\frac{-3750}{1--0.2}$	M1
	So, $S_\infty = -3125$	A1
		(2) [7]
	Notes	
(a)	B1: for both $ar = 750$ and $ar^4 = -6$ (may be implied from later working in either (a) or (b)). M1: **for eliminating *a*** by either dividing $ar^4 = -6$ by $ar = 750$ or dividing $ar = 750$ by $ar^4 = -6$, to achieve an equation in r^3 or $\frac{1}{r^3}$ Note that $r^4 - r = -\frac{6}{750}$ is M0. Note also that any of $r^3 = \frac{-6}{750}$ or $r^3 = \frac{750}{-6}\{=-125\}$ or $\frac{1}{r^3} = \frac{-6}{750}$ or $\frac{1}{r^3} = \frac{750}{-6}\{=-125\}$ are fine for the award of M1. SC: $ar^\alpha = 750$ and $ar^\beta = -6$ leading to $r^\delta = \frac{-6}{750}$ or $r^\delta = \frac{750}{-6}\{=-125\}$ or $\frac{1}{r^\delta} = \frac{-6}{750}$ or $\frac{1}{r^\delta} = \frac{750}{-6}\{=-125\}$ where $\delta = \beta - \alpha$ and $\delta \geq 2$ are fine for the award of M1. SC: $ar^2 = 750$ and $ar^5 = -6$ leading to $r = -\frac{1}{5}$ scores B0M1A1.	
(b)	M1 for inserting their r into either of their original correct equations of either $ar = 750$ or $\{a =\}\frac{750}{r}$ or $ar^4 = -6$ or $\{a =\}\frac{-6}{r^4}$ – in ***both a* and *r***. No slips allowed here for M1. A1 for either $a = -3750$ or a equal to the correct follow through result expressed either as an exact integer, or a fraction in the form $\frac{c}{d}$ where both c and d are integers, or correct to awrt 1 dp.	
(c)	M1 for applying $\frac{a}{1-r}$ correctly (only a slip in substituting r is allowed) using both their a and their $\|r\| < 1$. Eg. $\frac{-3750}{1--0.2}$. A1 for -3125 In parts (a) or (b) or (c), the correct answer with no working scores full marks.	

Q7.

Question Number	Scheme		Marks
(a)	$\{ar = 192 \text{ and } ar^2 = 144\}$		
	$r = \dfrac{144}{192}$	Attempt to eliminate a. (See notes.)	M1
	$r = \frac{3}{4}$ or 0.75	$\frac{3}{4}$ or 0.75	A1
			[2]
(b)	$a(0.75) = 192$		M1
	$a\left\{= \dfrac{192}{0.75}\right\} = 256$	256	A1
			[2]
(c)	$S_\infty = \dfrac{256}{1-0.75}$	Applies $\dfrac{a}{1-r}$ correctly using both their a and their $\|r\| < 1$.	M1
	So, $\{S_\infty =\}\ 1024$	1024	A1 cao
			[2]
(d)	$\dfrac{256(1-(0.75)^n)}{1-0.75} > 1000$	Applies S_n with their a and r and "uses" 1000 at any point in their working. (Allow with = or <).	M1
	$(0.75)^n < 1 - \dfrac{1000(0.25)}{256}\ \left\{= \dfrac{6}{256}\right\}$	Attempt to isolate $+(r)^n$ from S_n formula. (Allow with = or >).	M1
	$n\log(0.75) < \log\left(\dfrac{6}{256}\right)$	Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)	M1
	$n > \dfrac{\log\left(\frac{6}{256}\right)}{\log(0.75)} = 13.0471042... \Rightarrow n = 14$	See notes and $n = 14$	A1 cso
			[4]
			10

Q8.

Question Number	Scheme	Marks
(a)	Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$ Or one of: $r = \frac{k}{k+4},\ r = \frac{2k-15}{k},\ r^2 = \frac{2k-15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$	M1
	$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$	M1, A1
	Proceed to $k^2 - 7k - 60 = 0$ (*)	A1 (4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1 (2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k\text{-}15}{k} = \frac{12}{16}\left(= \frac{3}{4} \text{ or } 0.75\right)$	M1 A1 (2)
(d)	$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1 (2) **[10]**
(a)	**M1**: The 'initial step', scoring the first M mark, may be implied by next line of proof **M1**: Eliminates *a* and *r* to give valid equation in *k* only. Can be awarded for equation involving fractions. **A1** : need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation) **A1**: as answer is printed this mark is for cso (Needs = 0) **All four marks must be scored in part (a)**	
(b)	**M1**: Attempt to solve quadratic **A1**: This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen. Substitute and verify is **M1 A0** **Marks must be scored in part (b)**	
(c)	**M1**: Complete method to find *r* Could have answer in terms of *k* **A1**: 0.75 or any correct equivalent **Both Marks must be scored in (c)**	
(d)	**M1**: Tries to use $\frac{a}{1-r}$, (even with $r>1$). Could have an answer still in terms of *k*. **A1**: This answer is 64 cao.	

Q9.

Question Number	Scheme	Marks
(a)	$18000\times(0.8)^3 \quad =£9216$ * [may see $\frac{4}{5}$ or 80% or equivalent].	B1cso (1)
(b)	$18000\times(0.8)^n<1000$	M1
	$n\log(0.8)<\log(\frac{1}{18})$	M1
	$n>\dfrac{\log(\frac{1}{18})}{\log(0.8)}=12.952....$ so $n=13$.	A1 cso (3)
(c)	$u_5=200\times(1.12)^4$, $=£314.70$ or $£314.71$	M1, A1 (2)
(d)	$S_{15}=\dfrac{200(1.12^{15}-1)}{1.12-1}$ or $\dfrac{200(1-1.12^{15})}{1-1.12}$, = 7455.94….. awrt £7460	M1A1, A1 (3) [9]
(a)	B1 NB Answer is printed so **need working**. May see as above or ×0.8 in three steps giving 14400, 11520, 9216. Do not need to see £ sign but should see 9216 .	
(b)	1st M1 for an attempt to use *n*th term and 1000. Allow n or $n-1$ and allow > or = 2nd M1 for use of logs to find n Allow n or $n-1$ and allow > or = A1 Need $n=13$ This is an accuracy mark and must follow award of both M marks but should not follow incorrect work using $n-1$ for example. Condone slips in inequality signs here.	
(c)	M1 for use of their a and r in formula for 5th term of GP A1 cao need one of these answers – answer can imply method here NB 314.7 – A0	
(d)	M1 for use of sum to 15 terms of GP using their a and their r (allow if formula stated correctly and one error in substitution, but must use n not n - 1) 1st A1 for a fully correct expression (not evaluated)	
(b)	Alternative Methods Trial and Improvement See 989.56 (or 989 or 990) identified with 12, 13 or 14 years for first **M1** See 1236.95 (or 1236 or 1237) identified with 11, 12 or 13 years for second **M1** Then $n=13$ is **A1 (needs both Ms)** **Special case** $18000\times(0.8)^n<1000$ so $n=13$ as 989.56<1000 is M1M0A0 (not discounted $n=12$)	
(c)	May see the terms 224, 250.88, 280.99, 314.71 with a small slip for M1 A0, or done accurately for M1A1	
(d)	Adds 15 terms 200 + 224 + 250.88+… + (977.42) **M1** Seeing 977… is **A1** Obtains answer 7455.94 **A1** or awrt £7460 NOT 7450	

Q10.

Question number	Scheme	Marks
	(a) $50\,000r^{n-1}$ (or equiv.) (Allow ar^{n-1} if $50\,000r^{n-1}$ is seen in (b))	B1 (1)
	(b) $50\,000r^{n-1} > 200000$	M1
	(Using answer to (a), which must include r and n, and 200 000) (Allow equals sign or the wrong inequality sign) (Condone 'slips' such as omitting a zero)	
	$r^{n-1} > 4 \quad \Rightarrow \quad (n-1)\log r > \log 4$	M1
	(Introducing logs and dealing correctly with the power) (Allow equals sign or the wrong inequality sign)	
	$n > \dfrac{\log 4}{\log r} + 1$ (*)	A1cso (3)
	(c) $r = 1.09$: $n > \dfrac{\log 4}{\log 1.09} + 1$ or $n - 1 > \dfrac{\log 4}{\log 1.09}$ $(n > 17.086...)$ (Allow equality)	M1
	Year 18 or 2023 (If one of these is correct, ignore the other)	A1 (2)
	(d) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{50000(1-1.09^{10})}{1-1.09}$	M1 A1
	£760 000 (Must be this answer… nearest £10000)	A1 (3)
		9
	(b) <u>Incorrect</u> inequality sign at any stage loses the A mark. Condone missing brackets if otherwise correct, e.g $n-1\log r > \log 4$. <u>A common mistake:</u> $50\,000r^{n-1} > 200000$ M1 $(n-1)\log 50\,000r > \log 200000$ M0 ('Recovery' from here is not possible). (c) Correct answer with no working scores full marks. Year 17 (or 2022) with no working scores M1 A0. Treat other methods (e.g. "year by year" calculation) as if there is no working. (d) M1: Use of the correct formula with $a = 50000$, 5000 or 500000, and $n = 9$, 10, 11 or 15. M1 can also be scored by a "year by year" method, <u>with terms added</u>. (Allow the M mark if there is evidence of adding 9, 10, 11 or 15 terms). 1st A1 is scored if 10 correct terms have been added (allow "nearest £100"). (50000, 54500, 59405, 64751, 70579, 76931, 83855, 91402, 99628, 108595) <u>No</u> working shown: Special case: 760 000 scores 1 mark, scored as 1, 0, 0. (Other answers with no working score no marks).	

Q11.

Question Number	Scheme	Marks
(a)	$a = 4p,\ ar = (3p+15)$ and $ar^2 = 5p+20$	B1
	(So $r =$) $\frac{5p+20}{3p+15} = \frac{3p+15}{4p}$ or $4p(5p+20) = (3p+15)^2$ or equivalent	M1
	See $(3p+15)^2 = 9p^2 + 90p + 225$	M1
	$20p^2 + 80p = 9p^2 + 90p + 225 \rightarrow 11p^2 - 10p - 225 = 0$ *	A1 *
		(4)
(b)	$(p-5)(11p+45)$ so $p =$	M1
	$p = 5$ only (after rejecting - 45/11)	A1
	N.B. Special case p = 5 can be verified in (b) (1 mark only)	
	$11\times 5^2 - 10\times 5 - 225 = 275 - 50 - 225 = 0$ M1A0	
		(2)
(c)	$\frac{3\times 5+15}{4\times 5}$ or $\frac{5\times 5+20}{3\times 5+15}$	M1
	$r = \frac{3}{2}$	A1
		(2)
(d)	$S_{10} = \dfrac{20\left(1-\left("\frac{3}{2}"\right)^{10}\right)}{\left(1-"\frac{3}{2}"\right)}$	M1A1ft
	$(= 2266.601568\ldots) = 2267$	A1
		(3)
		Total 11