



Functions (Domain, Range and Inverse) Exam Questions (Sheet 2)

Q1.

The function f is defined by

$$f: x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}, x > 3.$

(4)

(b) Find the range of $f.$

(2)

(c) Find $f^{-1}(x).$ State the domain of this inverse function.

(3)

The function g is defined by

$$g: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}.$

(3)

(Total 12 marks)

Q2.

The function f is defined by

$$f: x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$

(4)

(b) Find $f^{-1}(x)$

(3)

(c) Find the domain of f^{-1}

(1)

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7},$ giving your answer in terms of $e.$

(4)

(Total 12 marks)

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Q3.

The functions f and g are defined by

$$f: x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$
$$g: x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1} . (2)

(b) Show that the composite function gf is

$$gf: x \mapsto \frac{8x^3 - 1}{1 - 2x^3}. \quad (4)$$

(c) Solve $gf(x) = 0$. (2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$. (5)

(Total 13 marks)

Q4. [Note: Parts (c) and (d) of this question require knowledge of the modulus function]

The functions f and g are defined by

$$f: x \mapsto \ln(2x-1), \quad x \in \mathbb{R}, \quad x > \frac{1}{2},$$
$$g: x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

(a) Find the exact value of $fg(4)$. (2)

(b) Find the inverse function $f^{-1}(x)$, stating its domain. (4)

(c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y -axis. (3)

(d) Find the exact values of x for which $\left| \frac{2}{x-3} \right| = 3$. (3)

(Total 12 marks)

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Q5. [Note: Part (c) of this question requires knowledge of the modulus function]

The function f is defined by

$$f: x \rightarrow \frac{3-2x}{x-5} \quad x \in \mathbb{R}, x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

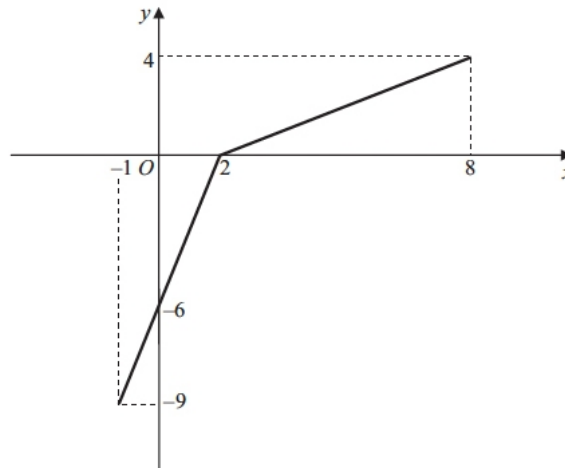


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

- (i) $y = |g(x)|$,
- (ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)

(Total 13 marks)



Q6. [Note: This question requires knowledge of the modulus function]

The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \quad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \quad x \in \mathbb{R}$$

(a) State the range of f .

(2)

(b) Find $fg(1)$.

(2)

(c) Find g^{-1} , the inverse function of g .

(2)

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)

(Total 11 marks)

Q7.

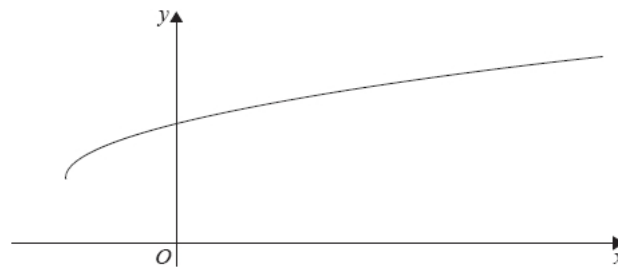


Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}, \quad x \geq -2$$

(a) State the range of g .

(1)

(b) Find $g^{-1}(x)$ and state its domain.

(3)

(c) Find the exact value of x for which

$$g(x) = x$$

(4)

(d) Hence state the value of a for which

$$g(a) = g^{-1}(a)$$

(1)

(Total for question = 9 marks)



Q8.

The function f is defined by

$$f : x \rightarrow e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of f . (1)

(b) Find f^{-1} and state its domain. (3)

The function g is defined by

$$g : x \rightarrow \ln(2x), \quad x > 0$$

(c) Solve the equation (4)

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

(d) Find $fg(x)$, giving your answer in its simplest form. (2)

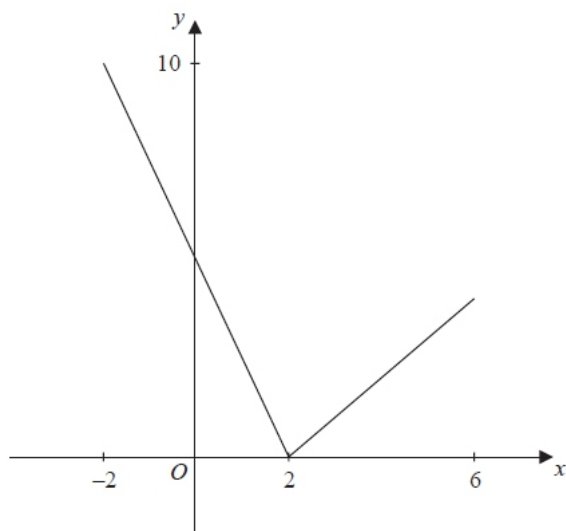
(e) Find, in terms of the constant k , the solution of the equation (2)

$$fg(x) = 2k^2$$

(Total 12 marks)

Q9.

The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in the diagram.



(a) Write down the range of f . (1)

(b) Find $ff(0)$. (2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(c) Find $g^{-1}(x)$ (3)

(d) Solve the equation $gf(x) = 16$ (5)

(Total 11 marks)