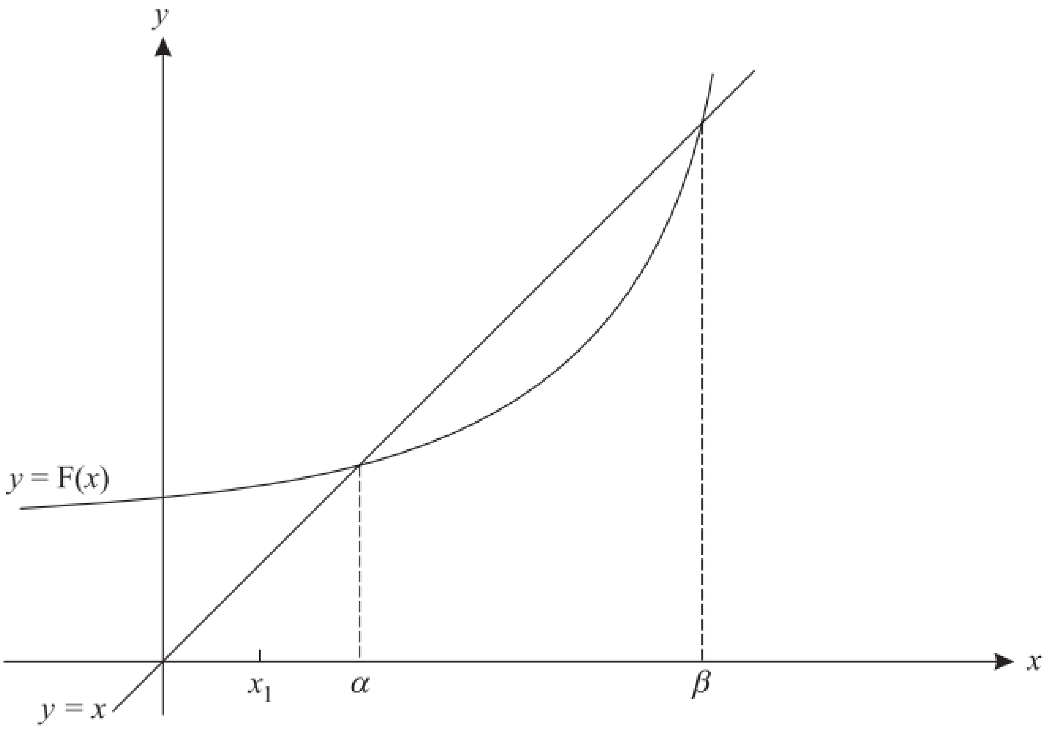
Fixed Point Iteration (From OCR 4726)

Note: For the questions that require inserts/answer sheets, these can be found at the end of this document.

Q1, (Jan 2006, Q4)

Answer the whole of this question on the insert provided.



The sketch shows the curve with equation $y = \mathrm{F}(x)$ and the line $y = x$. The equation $x = \mathrm{F}(x)$ has roots $x = \alpha$ and $x = \beta$ as shown.

(i) Use the copy of the sketch on the insert to show how an iteration of the form $x_{n+1} = \mathrm{F}(x_n)$, with starting value x_1 such that $0 < x_1 < \alpha$ as shown, converges to the root $x = \alpha$. [3]

(ii) State what happens in the iteration in the following two cases.

(a) x_1 is chosen such that $\alpha < x_1 < \beta$.

(b) x_1 is chosen such that $x_1 > \beta$.

[3]

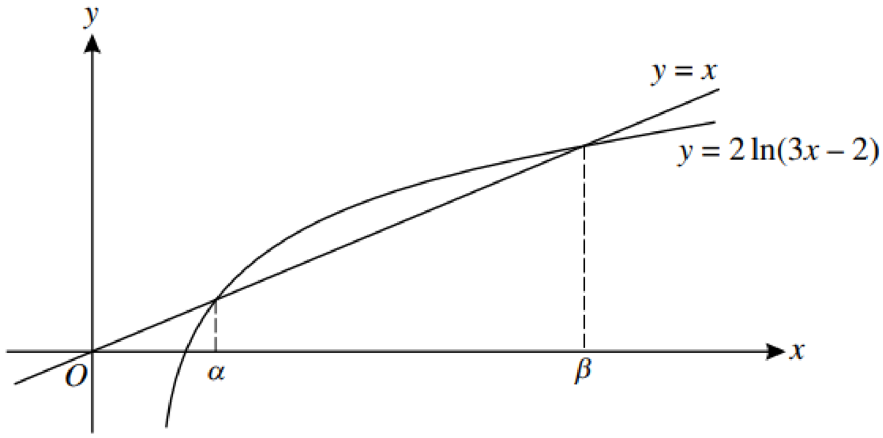
Q2, (Jun 2011, Q3i,iii)

It is given that $F(x) = 2 + \ln x$. The iteration $x_{n+1} = F(x_n)$ is to be used to find a root, α, of the equation $x = 2 + \ln x$.

(i) Taking $x_1 = 3.1$, find x_2 and x_3, giving your answers correct to 5 decimal places. **[2]**

(iii) Illustrate the iteration by drawing a sketch of $y = x$ and $y = F(x)$, showing how the values of x_n approach α. State whether the convergence is of the 'staircase' or 'cobweb' type. **[3]**

Q3, (Jun 2010 Q7i,ii)



The line $y = x$ and the curve $y = 2\ln(3x - 2)$ meet where $x = \alpha$ and $x = \beta$, as shown in the diagram.

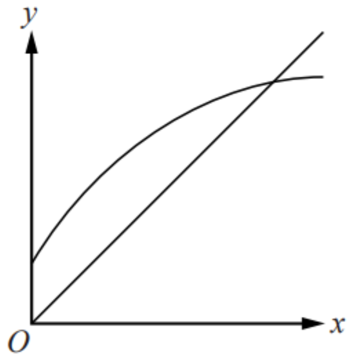
(i) Use the iteration $x_{n+1} = 2\ln(3x_n - 2)$, with initial value $x_1 = 5.25$, to find the value of β correct to 2 decimal places. Show all your working. **[2]**

(ii) With the help of a 'staircase' diagram, explain why this iteration will not converge to α, whatever value of x_1 (other than α) is used. **[3]**

Q4, (Jun 2012, Q4)

It is given that the equation $x^4 - 2x - 1 = 0$ has only one positive root, α, and $1.3 < \alpha < 1.5$.

(i)



The diagram shows a sketch of $y = x$ and $y = \sqrt[4]{2x+1}$ for $x \geqslant 0$. Use the iteration $x_{n+1} = \sqrt[4]{2x_n + 1}$ with $x_1 = 1.35$ to find x_2 and x_3, correct to 4 decimal places. On the copy of the diagram show how the iteration converges to α. **[3]**

(ii) For the same equation, the iteration $x_{n+1} = \frac{1}{2}\left(x_n^4 - 1\right)$ with $x_1 = 1.35$ gives $x_2 = 1.1608$ and $x_3 = 0.4077$, correct to 4 decimal places. Draw a sketch of $y = x$ and $y = \frac{1}{2}\left(x^4 - 1\right)$ for $x \geqslant 0$, and show how this iteration does not converge to α. **[2]**

(iii) Find the positive root of the equation $x^4 - 2x - 1 = 0$ by using the Newton-Raphson method with $x_1 = 1.35$, giving the root correct to 4 decimal places. **[4]**

Q5, (Jan 2013, Q8i)

It is required to solve the equation $\ln(x - 1) - x + 3 = 0$.

You are given that there are two roots, α and β, where $1.1 < \alpha < 1.2$ and $4.1 < \beta < 4.2$.

(i) The root β can be found using the iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3.$$

(a) Using this iterative formula with $x_1 = 4.15$, find β correct to 3 decimal places. Show all your working. **[2]**

(b) Explain with the aid of a sketch why this iterative formula will not converge to α whatever initial value is taken. **[3]**

Q6, (Jun 2014, Q9i,ii)

The equation $10x - 8\ln x = 28$ has a root α in the interval [3, 4]. The iteration $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x) = 2.8 + 0.8\ln x$ and $x_1 = 3.8$, is to be used to find α.

(i) Find the value of α correct to 5 decimal places. You should show the result of each step of the iteration to 6 decimal places. **[4]**

(ii) Illustrate this iteration by means of a sketch. **[2]**

Q6, (Jun 2016, Q4)

You are given the equation $(2x-1)^2 - \mathrm{e}^x = 0$.

(i) Verify that 0 is a root of the equation. **[1]**

There are also two other roots, α and β, where $0 < \alpha < \beta$.

(ii) The iterative formula $x_{r+1} = \ln(2x_r - 1)^2$ is to be used to find a root of the equation.

(a) Sketch the line $y = x$ and the curve $y = \ln(2x-1)^2$ on the same axes, showing the roots 0, α and β. **[3]**

(b) By drawing a 'staircase' diagram on your sketch, starting with a value of x that is between α and β, show that this iteration does not converge to α. **[1]**

(c) Using this iterative formula with $x_1 = 3.75$, find the value of β correct to 3 decimal places. **[3]**

(iii) Using the Newton-Raphson method with $x_1 = 1.6$, find the root α of the equation $(2x-1)^2 - \mathrm{e}^x = 0$ correct to 5 significant figures. Show the result of each iteration. **[4]**

Inserts and Answer Sheets

Q1.

(i)