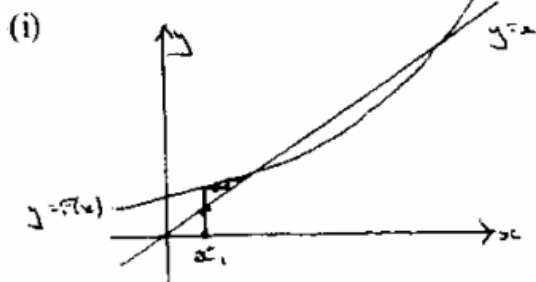


Fixed Point Iteration (From OCR 4726)

Note: For the questions that require inserts/answer sheets, these can be found at the end of this document.

Q1, (Jan 2006, Q4)



B1 Line from x_1 to curve

B1 Then to line

B1 Clear explanation; allow use of step/staircase

- (ii)(a) Converges to $x=\alpha$
- (b) Diverges (does not give either root)

B1, B1

B1

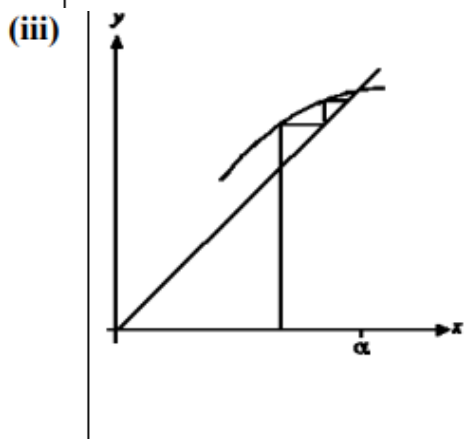
Q2, (Jun 2011, Q3i,iii)

- (i) $x_1 = 3.1 \Rightarrow x_2 = 3.13140,$
 $x_3 = 3.14148$

B1 For correct x_2

B1 For correct x_3

2



B1 For $y = x$ and $y = F(x)$ drawn, crossing as shown

B1 For lines drawn to illustrate iteration (Min 2 horizontal and 2 vertical seen)

Staircase

B1 For stating "staircase"

3

Q3, (Jun 2010 Q7i,ii)

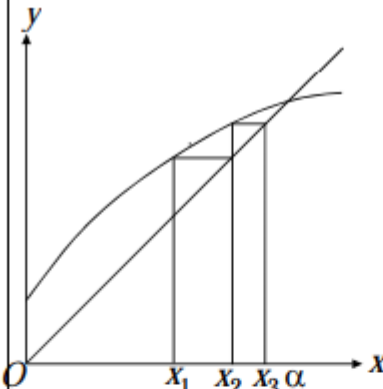
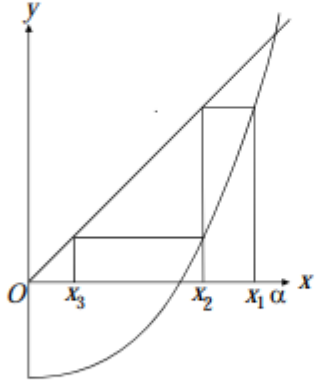
- (i) Get 5.242, 5.239, 5.237
Get 5.24

B1√ Any 3(minimum) correct from previous value

B1 Allow one B1 for 5.24 seen if 2 d.p.used

- (ii) Show reasonable staircase for any region B1 Drawn curve to line
- Describe any one of the three cases B1
- Describe all three cases B1

Q4, (Jun 2012, Q4)

| | | | |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(i)</p> |  <p> $x_2 = 1.3869\dots$ $x_3 = 1.3938$ </p> | <p>B1 B1 B1</p> | <p>For correct value (4 d.p. or better) For correct value. For sketch showing staircase towards α. (Vertical lines do not need to be labelled)</p> |
| <p>(ii)</p> |  | <p>B1 B1</p> | <p>For sketch like $y = \frac{1}{2}(x^4 - 1)$ and $y = x$ (curve or continuation of curve cuts - y axis.) For sketch showing staircase away from α. ("Away" means labelling or arrows required.) Labelling means x_1, x_2, \dots in right place or numeric values.</p> |
| <p>(iii)</p> | $x_{n+1} = x_n - \frac{x_n^4 - 2x_n - 1}{4x_n^3 - 2}$ <p> $1.35 \rightarrow 1.398268$ $\rightarrow 1.395348 \rightarrow 1.395337$ $\Rightarrow 1.3953$ </p> | <p>M1 A1 A1 A1</p> | <p>For deriving the iterative formula For correct formula For 1st value For correct 4dp α with 2 iterates equal to 4 dp. (i.e. last two iterates agree to 4dp) www</p> |

[3]

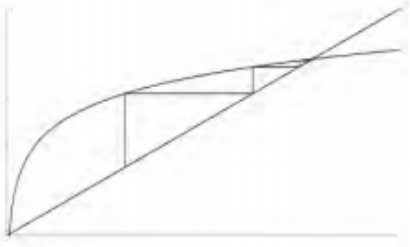
[2]

[4]

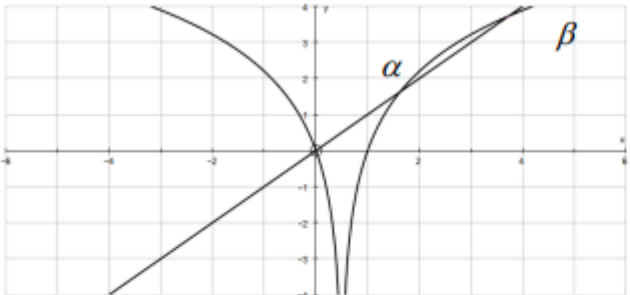
Q5, (Jan 2013, Q8i)

| | | | | | |
|------------|------------|----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|-----------------------------------------|
| (i) | (a) | $x_1 = 4.15, \quad x_2 = 4.1474\dots$ $x_3 = 4.1465\dots, \quad x_4 = 4.1463\dots$ $\beta = 4.146$ | M1 A1 [2] | Using iterative formula at least once using at least 4dp www | All iterates must be seen |
| (i) | (b) | Staircase diagram will always move to upper root | B1 B1 B1 [3] | Sketch showing an example $x_1 > \alpha$ Example with $x_1 < \alpha$ Statement Dep on 1st two B | Ignore any statement when $x_1 > \beta$ |

Q6, (Jun 2014, Q9i,ii)

| | | | | | | | | | | | | | | | | | | | | |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|--------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------------|--|------------------------------------------------------------------------------------------------|------------------------|---------------------------|
| (i) | <table border="0"> <tr> <td>3.8</td> <td>3.868001</td> </tr> <tr> <td>3.868001</td> <td>3.882190</td> </tr> <tr> <td>3.882190</td> <td>3.885120</td> </tr> <tr> <td>3.885120</td> <td>3.885723</td> </tr> <tr> <td>3.885723</td> <td>3.885847</td> </tr> <tr> <td>3.885847</td> <td>3.885873</td> </tr> <tr> <td>3.885873</td> <td>3.885878</td> </tr> <tr> <td colspan="2">Root = 3.88588</td> </tr> </table> | 3.8 | 3.868001 | 3.868001 | 3.882190 | 3.882190 | 3.885120 | 3.885120 | 3.885723 | 3.885723 | 3.885847 | 3.885847 | 3.885873 | 3.885873 | 3.885878 | Root = 3.88588 | | M1 A1 A1 A1 [4] | For x_2 For x_3 | N.B. Working must be seen |
| 3.8 | 3.868001 | | | | | | | | | | | | | | | | | | | |
| 3.868001 | 3.882190 | | | | | | | | | | | | | | | | | | | |
| 3.882190 | 3.885120 | | | | | | | | | | | | | | | | | | | |
| 3.885120 | 3.885723 | | | | | | | | | | | | | | | | | | | |
| 3.885723 | 3.885847 | | | | | | | | | | | | | | | | | | | |
| 3.885847 | 3.885873 | | | | | | | | | | | | | | | | | | | |
| 3.885873 | 3.885878 | | | | | | | | | | | | | | | | | | | |
| Root = 3.88588 | | | | | | | | | | | | | | | | | | | | |
| (ii) |  | B1 B1 [2] | Curve and line Iterations showing staircase from below. At least two seen | Concave curve initially above $y = x$ Only [3,4] required so ignore behaviour at origin | | | | | | | | | | | | | | | | |

Q6, (Jun 2016, Q4)

| | | | | | |
|-------|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------|
| (i) | | $x = 0$ in equation satisfies as $e^0 = 1$. | B1 [1] | | |
| (ii) | (a) |  | B1 B1 B1 [3] | Asymptote between $x = 0$ and where it crosses x axis . +ve roots clear LH branch going through origin LH branch does not have to be complete | Allow one branch. SC1 $y = (\ln(2x - 1))^2$ |
| | (b) | Staircase seen near middle root to be converging to β . | B1 [1] | Either starting point shown with vertical line from axis to curve or arrows on staircase lines | Follow through their curve where there are two positive roots |
| (ii) | (c) | $x_1 = 3.75$ $x_2 = 3.743604\dots$ Leading to 3.733 | B3 [3] | For correct answer B2 for 3.734 B1 for sight of 3.7436... | |
| (iii) | | $f(x) = (2x - 1)^2 - e^x$ $\Rightarrow f'(x) = 4(2x - 1) - e^x$ $\Rightarrow x_{r+1} = x_r - \frac{(2x - 1)^2 - e^x}{4(2x - 1) - e^x}$ $\Rightarrow x_2 = 1.629382\dots, x_3 = 1.629053$ Root = 1.6291 | B1 M1 A1 A1 [4] | $f'(x)$ correct soi by x_2 Use of formula soi by x_2 x_2 to 2dp or better Correct root stated to 5sf | $f(x)$ correct and their $f'(x)$ At least 2 iterates shown |