



Fixed Point Iteration Exam Questions (Edexcel)

Q1.

$$f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm \frac{4}{5} e^{-x}$

(1)

The equation $f(x) = 0$ has a root α , where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5} e^{-x_n}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

(Total 11 marks)

(Q24 6665/01, June 2013)

Q2.

A curve C has equation $y = e^{4x} + x^4 + 8x + 5$

(a) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$

(3)

(b) On the axes given on page 5, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y -axis and state the equation of any asymptotes.

(4)

(c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

(1)

The iteration formula

$$x_{n+1} = (-2 - e^{4x_n})^{\frac{1}{3}}, \quad x_0 = -1$$

can be used to find an approximate value for this root.

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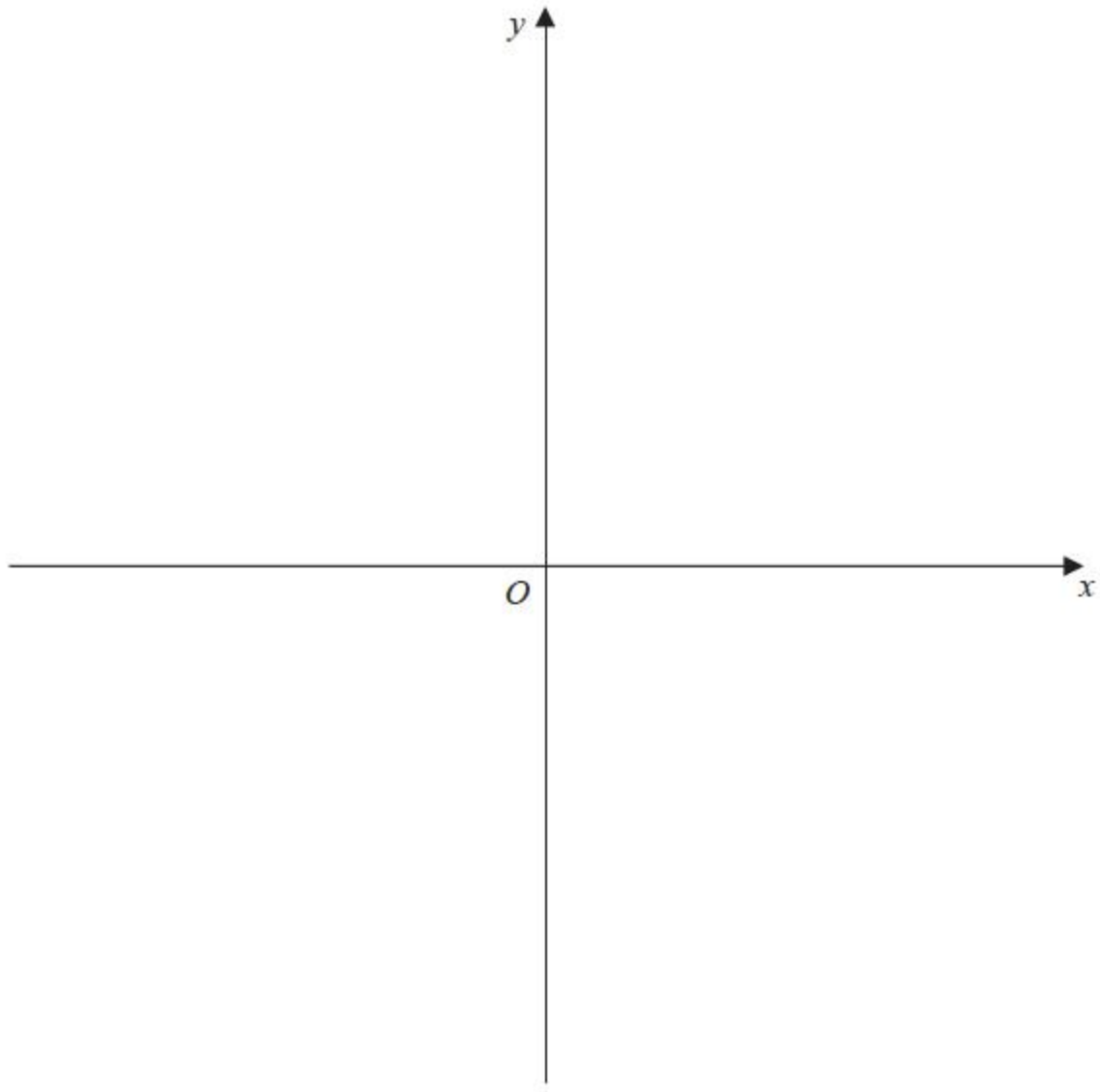


(d) Calculate the values of x_1 and x_2 , giving your answers to 5 decimal places.

(2)

(e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C .

(2)



(Total 12 marks)

(Q21 6665/01/R, June 2014)

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Q3.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P .

(a) Find the exact coordinates of P .

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1 , x_2 and x_3 .

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

(Total 11 marks)

(Q25 6665/01, Jan 2009)

Q4.

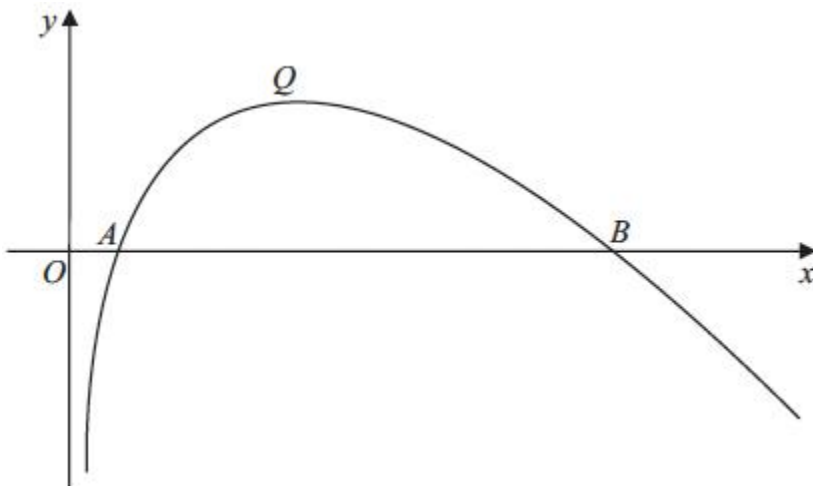


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \quad x > 0$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B .

(2)

(b) Find $f'(x)$.

(3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6

(2)



(d) Show that the x -coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}$$

(3)

To find an approximation for the x -coordinate of Q , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places.

(3)

(Total 13 marks)

(Q24 6665/01, Jan 2011)

Q5.

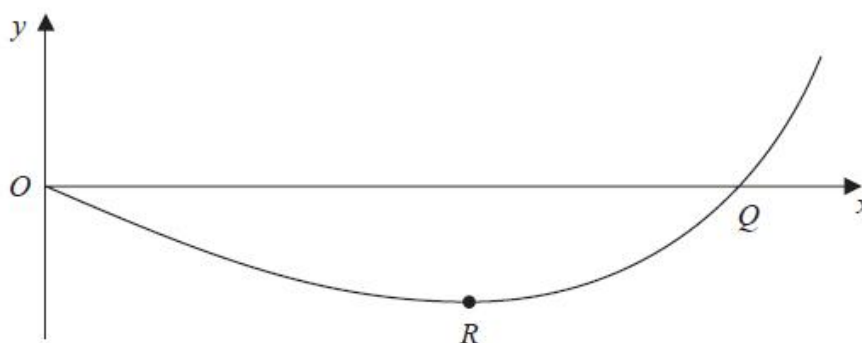


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x -axis at the point Q and has a minimum turning point at R .

(a) Show that the x coordinate of Q lies between 2.1 and 2.2

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x \sin\left(\frac{1}{2}x^2\right)}$$

(4)



Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n \sin\left(\frac{1}{2}x_n^2\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

(Total 8 marks)

(Q26 6665/01, June 2014)

Q6.

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{3+x}\right)}, \quad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{3+x_n}\right)}, \quad n \geq 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1 , x_2 , and x_3 .

(3)

The root of $f(x) = 0$ is α .

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

(Total 9 marks)

(Q21 6665/01, June 2012)



Q7.

$$f(x) = x^2 - 3x + 2 \cos\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P .

(b) Show that the x -coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x -coordinate of P is 1.9078 correct to 4 decimal places.

(3)

(Total 12 marks)

(Q23 6665/01, Jan 2012)

Q8.

$f(x) = 4\operatorname{cosec} x - 4x + 1$, where x is in radians.

(a) Show that there is a root α of $f(x) = 0$ in the interval $[1.2, 1.3]$.

(2)

(b) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 4 decimal places.

(3)

(d) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

(Total 9 marks)

(Q22 6665/01, June 2010)