Fixed Point Iteration Mark Scheme (Edexcel)

Q1.

Question Number	Scheme	Marks
(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe.	M1A1
	Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$	M1A1
	$\Rightarrow x_2 = \text{awrt } 0.492,\ x_3 = \text{awrt } 0.489$	A1
		(3)
(d)	$\alpha = 0.49$	B1
	$f(0.485) = -0.487,\ f(0.495) = (+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu' + uv'$. If the rule is quoted it must be correct.
It can be implied by their $u = ..., v = ..., u' = ..., v' = ...$ followed by their $vu' + uv'$
If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x} + Bxe^{2x}$

A1 $f'(x) = 50x^2e^{2x} + 50xe^{2x}$.
Allow un simplified forms such as $f'(x) = 25x^2 \times 2e^{2x} + 50x \times e^{2x}$

dM1 Sets $f'(x) = 0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x
This is dependent upon the first M1 being scored.

A1 Both $x = -1$ and $x = 0$ or one complete coordinate . Accept $(0,-16)$ and $(-1, 25e^{-2}-16)$ or $(-1, awrt -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.
$x = -1, 0 \quad y = \frac{25}{e^2} - 16, -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1, 25e^{-2}-16)$ but the 'pairs' must be correct and exact.

Notes for Question Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x}-16=0$ or $25x^2e^{2x}=16$

2) Show at least one intermediate (correct) line with either

x^2 or x the subject. Eg $x^2=\frac{16}{25}e^{-2x}$, $x=\sqrt{\frac{16}{25}e^{-2x}}$ oe

or square rooting $25x^2e^{2x}=16 \Rightarrow 5xe^x=\pm 4$

or factorising by DOTS to give $(5xe^x+4)(5xe^x-4)=0$

3) Show the given answer $x=\pm\frac{4}{5}e^{-x}$.

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that f(x)=0

(c)

M1 Substitutes $x_0=0.5$ into $x=\frac{4}{5}e^{-x} \Rightarrow x_1=....$

This can be implied by $x_1=\frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 $x_1=$ awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript.

A1 $x_2=$ awrt 0.492, $x_3=$ awrt 0.489 3dp. Mark as the second and third values given.

(d)

B1 States $\alpha=0.49$

B1 Justifies by

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$f(0.485)=-0.5$, $f(0.495)=(+)0.5$ rounded

$f(0.485)=-0.4$, $f(0.495)=(+)0.4$ truncated

giving a reason – accept change of sign, >0 <0 or $f(0.485)\times f(0.495)<0$

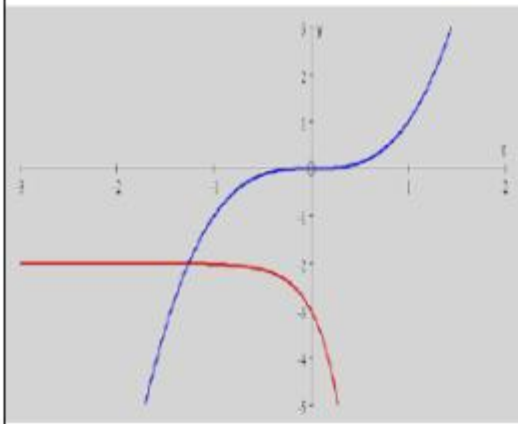
and giving a minimal conclusion. Eg. Accept hence root or $\alpha=0.49$

A smaller interval containing the root may be used, eg $f(0.49)$ and $f(0.495)$. Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of $x_4=$ awrt 0.491 and stating (or seeing each value round to) 0.49

Q2.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$	M1, A1
	Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$	A1 *
		(3)
(b)		
	$y = x^3$	B1
	Shape of $y = -2 - e^{4x}$	B1
	$y = -2 - e^{4x}$ cuts y axis at (0,-3)	B1
	$y = -2 - e^{4x}$ has asymptote at $y = -2$	B1
		(4)
(c)	Only one crossing point	B1
		(1)
(d)	−1.26376, -1.26126 Accept answers which round to these answers to 5dp	M1 A1 (2)
(e)	α = -1.26 and so turning point is at (-1.26, -2.55)	M1 A1cao (2)
		12 marks

(a)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$

A1* States or sets $\frac{dy}{dx} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - e^{4x}$.

(b)

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin. It must only appear in Quadrants 1 and 3 and have a gradient that is always $\geqslant 0$. The gradient should appear large at either end. Tolerate slips of the pen.See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - e^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - e^{4x}$ cutting or meeting the y axis at (0,-3). Its shape is not important. Accept for the intention of (0,-3), -3 being marked on the y – axis as well as (-3,0)
Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - e^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as '-2' or indeed $x = -2$. See practice and qualification for acceptable solutions.

(c)

B1 Score for a statement to the effect that the graphs cross at one point. Accept minimal statements such as 'one intersection'. Do not award if their diagram shows more than one intersection. They must have a diagram (which may be incorrect)

(d)

M1 Awarded for applying the iteration formula once. Possible ways in which this can be scored are the sight of $\sqrt[3]{-2 - e^{-4}}$, $(-2 - e^{4\times -1})^{\frac{1}{3}}$ or awrt -1.264

A1 Both values correct awrt -1.26376, -1.26126 5dps. The subscripts are unimportant for this mark. Score as the first and second values seen.

(e)

M1 Score for EITHER rounding their value in part (c) to 2 dp OR finding turning point of C by substituting a value of x generated from part (d) into $y = e^{4x} + x^4 + 8x + 5$ in order to find the y value. You may accept the appearance of a y value as evidence of finding the turning point (as long as an x value appears to be generated from part (d) and the correct equation is used.)

A1 (-1.26, -2.55) and correct solution only. It is a deduction and you cannot accept the appearance of a correct answer for two marks.

Q3.

Question Number	Scheme	Marks	
(a)	$f'(x) = 3e^x + 3xe^x$	M1 A1	
	$3e^x + 3xe^x = 3e^x(1+x) = 0$		
	$x = -1$	M1 A1	
	$f(-1) = -3e^{-1} - 1$	B1	(5)
(b)	$x_1 = 0.2596$	B1	
	$x_2 = 0.2571$	B1	
	$x_3 = 0.2578$	B1	(3)
(c)	Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1	
	$f(0.257\,55) = -0.000\,379\ ...$		
	$f(0.257\,65) = 0.000\,109\ ...$	A1	
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ✱ cso	A1	
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)		(3)
			[11]
	Note: $x = 0.257\,627\,65\ ...$ is accurate		

Q4.

Question Number	Scheme		Marks
(a)	Crosses x-axis $\Rightarrow f(x)=0 \Rightarrow (8-x)\ln x = 0$		
	Either $(8-x)=0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
(b)	Apply product rule: $\begin{cases} u=(8-x) & v=\ln x \\ \frac{du}{dx}=-1 & \frac{dv}{dx}=\frac{1}{x} \end{cases}$	$vu'+uv'$	M1
	$f'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$f'(3.5) = 0.032951317...$ $f'(3.6) = -0.058711623...$ Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $f'(3.5)$ and $f'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $f'(x)=0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $f'(x)=0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)
Question Number	**Scheme**		**Marks**
(e)	Iterative formula: $x_{n+1} = \frac{8}{\ln x_n + 1}$		
	$x_1 = \frac{8}{\ln(3.55)+1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both x_1 = awrt 3.529 and x_2 = awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534$, to 3 dp.	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3) [13]

Q5.

Question Number	Scheme	Marks
(a)	$y_{21} = -0.224$, $y_{22} = (+)0.546$	M1
	Change of sign$\Rightarrow Q$ lies between	A1
		(2)
(b)	At R $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$	A1
		(2)
		(8 marks)

(a)

M1 Sub both $x = 2.1$ and $x = 2.2$ into y and achieve at least one correct to 1 sig fig

In radians $y_{2.1} = \text{awrt} -0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

In degrees $y_{2.1} = \text{awrt } 3$ $y_{2.2} = \text{awrt } 4$

A1 Both values correct to 1 sf with a reason and a minimal conclusion.

$y_{2.1} = \text{awrt} -0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

Accept change of sign, positive and negative, $y_{2.1} \times y_{2.2} = -1$ as reasons and hence root, Q lies between 2.1 and 2.2, QED as a minimal conclusion.

Accept a smaller interval spanning the root of 2.131528, say 2.13 and 2.14, but the A1 can only be scored when the candidate refers back to the question, stating that as root lies between 2.13 and 2.14 it lies between 2.1 and 2.2

(b)

M1 Differentiating to get $\frac{dy}{dx} = \ldots \sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ where ... is a constant, or a linear function in x.

A1 $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$

M1 Sets their $\frac{dy}{dx} = 0$ and proceeds to make the x of their $3x^2$ the subject of the formula

Alternatively they could state $\frac{dy}{dx} = 0$ and write a line such as

$2x\sin\left(\frac{1}{2}x^2\right) = 3x^2 - 3$, before making the x of $3x^2$ the subject of the formula

A1* Correct given solution. $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$

Watch for missing x's in their formula

(c)

M1 Subs $x = 1.3$ into the iterative formula to find at least x_1.

This can be implied by $x_1 = \text{awrt } 1.3$ (not just 1.3)

or $x_1 = \sqrt{1 + \frac{2}{3} \times 1.3 \sin\left(\frac{1}{2} \times 1.3^2\right)}$ or $x_1 = \text{awrt } 1.006$ (degrees)

A1 Both answers correct (awrt 3 decimal places). The subscripts are not important. Mark as the first and second values seen. $x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$

Q6.

Question Number	Scheme	Marks
	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1A1*
		(3)
	(b) $x_1=1.41$, awrt $x_2=1.20$ $x_3=1.31$	M1A1,A1
		(3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	f(1.2725) = (+)0.00827... f(1.2715) = −0.00821....	M1
	Change of sign⇒α=1.272	A1
		(3)
		(9 marks)

Notes

(a) M1 Moves from f(x)=0, which may be implied by subsequent working, to $x^2(x\pm3)=\pm12\pm4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1 Divides by '(x+3)' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage

A1* CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12-4x needs to have been factorised.

(b) **Note that this appears B1,B1,B1 on EPEN**

M1 An attempt to substitute $x_0=1$ into the iterative formula to calculate x_1.

This can be awarded for the sight of $\sqrt{\dfrac{4(3-1)}{(3+1)}}$, $\sqrt{\dfrac{8}{4}}$, $\sqrt{2}$ and even 1.4

A1 $x_1=1.41$. The subscript is not important. Mark as the first value found. $\sqrt{2}$ is A0

A1 $x_2=$ awrt 1.20 $x_3=$ awrt 1.31. Mark as the second and third values found. Condone 1.2 for x_2

(c) **Note that this appears M1A1A1 on EPEN**

M1 Choosing the interval (1.2715,1.2725) or tighter containing the root 1.271998323. Continued iteration is not allowed for this question and is M0

M1 Calculates f(1.2715) and f(1.2725), or the tighter interval with at least 1 correct to 1 sig fig rounded or truncated.

Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp

Accept f(1.2725) = (+)0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1 Both values correct (see above).

A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0

And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

Alternative to (a) working backwards

(a)

	$x=\sqrt{\frac{4(3-x)}{(x+3)}} \Rightarrow x^2=\frac{4(3-x)}{(x+3)} \Rightarrow x^2(x+3)=4(3-x)$	M1
	$x^3+3x^2=12-4x \Rightarrow x^3+3x^2+4x-12=0$	dM1
	States that this is f(*x*)=0	A1* (3)

Alternative starting with the given result and working backwards

M1 Square (both sides) and multiply by $(x+3)$

dM1 Expand brackets and collect terms on one side of the equation =0

A1 A statement to the effect that this is f(*x*)=0

An acceptable answer to (c) with an example of a tighter interval

M1 Choosing the interval (1.2715, 1.2720). This contains the root 1.2719(98323)

M1 Calculates f(1.2715) and f(1.2720), with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated f(1.2715) = -0.01 2dp
Accept f(1.2720) = (+)0.00003 1sf rounded or f(1.2720) = (+)0.00002 truncated 1sf

A1 Both values correct (see above).
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2720)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

x	f(x)
1.2715	-0.00821362
1.2716	-0.00656564
1.2717	-0.00491752
1.2718	-0.00326927
1.2719	-0.00162088
1.2720	+0.00002765
1.2721	+0.00167631
1.2722	+0.00332511
1.2723	+0.00497405
1.2724	+0.00662312
1.2725	+0.00827233

An acceptable answer to (c) using g(x) where g(x)= $\sqrt{\frac{4(3-x)}{(x+3)}}-x$

2nd M1 Calculates g(1.2715) and g(1.2725), or the tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
g(1.2715) = 0.0007559. Accept g(1.2715) = awrt (+)0.0008 1sf rounded or awrt 0.0007 truncated.
g(1.2725)=-0.00076105. Accept g(1.2725) = awrt -0.0008 1sf rounded or awrt -0.0007 truncated.

Q7.

Question No	Scheme	Marks
6	(a) f(0.8) = 0.082, f(0.9)= -0.089 Change of sign $\Rightarrow$ root (0.8,0.9)	M1 A1 **(2)**
	(b) $f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$ Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$	M1 A1 M1A1* **(4)**
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$ x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	M1 A1,A1 **(3)**
	(d) [1.90775,1.90785] f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076.. Change of sign $\Rightarrow$ x=1.9078	M1 M1 A1 **(3)** **(12 marks)**

Q8.

Question Number	Scheme	Marks
(a)	$f(1.2) = 0.49166551...,\quad f(1.3) = -0.048719817...$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1 (2)
(b)	$4\text{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\text{cosec}x + 1$ $\Rightarrow x = \text{cosec}x + \frac{1}{4} \Rightarrow x = \frac{1}{\sin x} + \frac{1}{4}$	M1 A1 * (2)
(c)	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$ $x_1 = 1.303757858...,\quad x_2 = 1.286745793...$ $x_3 = 1.291744613...$	M1 A1 A1 (3)
(d)	$f(1.2905) = 0.00044566695...,\quad f(1.2915) = -0.00475017278...$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	M1 A1 (2) [9]
	(a) M1: Attempts to evaluate both f(1.2) and f(1.3) and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $f(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by x_1 = awrt 1.3 or x_1 = awrt 46°. A1: Both x_1 = awrt 1.3038 and x_2 = awrt 1.2867 A1: x_3 = awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	