Differentiation (Chain, Product and Quotient Rules) Introductory Exam Questions (From OCR 4723)

Q1, (Jan 2006, Q3)

(a) Differentiate $x^2(x+1)^6$ with respect to x. [3]

(b) Find the gradient of the curve $y = \dfrac{x^2+3}{x^2-3}$ at the point where $x = 1$. [3]

Q2, (Jun 2006, Q1)

Find the equation of the tangent to the curve $y = \sqrt{4x+1}$ at the point $(2, 3)$. [5]

Q3, (Jan 2007, Q1)

Find the equation of the tangent to the curve $y = \dfrac{2x+1}{3x-1}$ at the point $\left(1, \frac{3}{2}\right)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

Q4, (Jun 2007, Q1)

Differentiate each of the following with respect to x.

(i) $x^3(x+1)^5$ [2]

(ii) $\sqrt{3x^4+1}$ [3]

Q5, (Jun 2008, Q3)

Find, in the form $y = mx + c$, the equation of the tangent to the curve

$$y = x^2 \ln x$$

at the point with x-coordinate e. [6]

Q6, (Jan 2010, Q5)

The equation of a curve is $y = (x^2+1)^8$.

(i) Find an expression for $\dfrac{dy}{dx}$ and hence show that the only stationary point on the curve is the point for which $x = 0$. **[4]**

(ii) Find an expression for $\dfrac{d^2y}{dx^2}$ and hence find the value of $\dfrac{d^2y}{dx^2}$ at the stationary point. **[5]**

Q7, (Jun 2010, Q1)

Find $\dfrac{dy}{dx}$ in each of the following cases:

(i) $y = x^3e^{2x}$, **[2]**

(ii) $y = \ln(3+2x^2)$, **[2]**

(iii) $y = \dfrac{x}{2x+1}$. **[2]**

Q8, (Jan 2013, Q1)

For each of the following curves, find the gradient at the point with x-coordinate 2.

(i) $y = \dfrac{3x}{2x+1}$ **[3]**

(ii) $y = \sqrt{4x^2+9}$ **[3]**

Q9, (Jan 2013, Q4)

The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by

$$m = 250e^{0.021t}.$$

(i) Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value. **[3]**

(ii) Find the rate at which the mass is increasing at the instant when the mass is 400 grams. **[3]**

Q10, (Jun 2014, Q1)

Given that $y = 4x^2 \ln x$, find the value of $\dfrac{d^2y}{dx^2}$ when $x = e^2$. **[5]**

Q11, (Jun 2015, Q1)

Find the equation of the tangent to the curve $y = \dfrac{5x+4}{3x-8}$ at the point $(2, -7)$. **[5]**

Q12, (Jun 2016, Q1)

Find the equation of the tangent to the curve

$$y = 3x^2(x+2)^6$$

at the point $(-1, 3)$, giving your answer in the form $y = mx + c$. **[5]**

Q13, (OCR 4753, Jan 2009, Q2i)

Differentiate $x \cos 2x$ with respect to x. **[3]**

Q14, (OCR 4753, Jan 2012, Q1)

Differentiate $x^2 \tan 2x$. **[3]**

Q15, (OCR 4753, Jun 2010, Q3)

(i) Differentiate $\sqrt{1+3x^2}$. **[3]**

(ii) Hence show that the derivative of $x\sqrt{1+3x^2}$ is $\dfrac{1+6x^2}{\sqrt{1+3x^2}}$. **[4]**
