Differentiation (Chain, Product and Quotient Rules) Introductory Exam Questions (From OCR 4723)

Q1, (Jan 2006, Q3)

(a)	Attempt use of product rule	**M1**		[involving … + …]
	Obtain $2x(x+1)^6$ …	**A1**		
	Obtain … + $6x^2(x+1)^5$	**A1**	**3**	[or equivs; ignore subsequent attempt at simplification]
(b)	Attempt use of quotient rule	**M1**		[or, with adjustment, product rule; allow u / v confusion]
	Obtain $\dfrac{(x^2-3)2x-(x^2+3)2x}{(x^2-3)^2}$	**A1**		[or equiv]
	Obtain -3	**A1**	**3**	[from correct derivative only]

Q2, (Jun 2006, Q1)

Differentiate to obtain $k(4x+1)^{-\frac{1}{2}}$	**M1**		any non-zero constant k
Obtain $2(4x+1)^{-\frac{1}{2}}$	**A1**		or equiv, perhaps unsimplified
Obtain $\frac{2}{3}$ for value of first derivative	**A1**		or unsimplified equiv
Attempt equation of tangent through (2, 3)	**M1**		using numerical value of first derivative provided derivative is of form $k'(4x+1)^n$
Obtain $y=\frac{2}{3}x+\frac{5}{3}$ or $2x-3y+5=0$	**A1**	**5**	or equiv involving 3 terms

Q3, (Jan 2007, Q1)

Attempt use of quotient rule to find derivative	M1		allow for numerator 'wrong way round'; or attempt use of product rule
Obtain $\dfrac{2(3x-1)-3(2x+1)}{(3x-1)^2}$	A1		or equiv
Obtain $-\frac{5}{4}$ for gradient	A1		or equiv
Attempt eqn of straight line with numerical gradient	M1		obtained from their $\frac{dy}{dx}$; tangent not normal
Obtain $5x+4y-11=0$	A1	**5**	or similar equiv

Q4, (Jun 2007, Q1)

(i)	Attempt use of product rule	M1		
	Obtain $3x^2(x+1)^5+5x^3(x+1)^4$	A1	**2**	or equiv
	[Or: (following complete expansion and differentiation term by term)			
	Obtain $8x^7+35x^6+60x^5+50x^4+20x^3+3x^2$	B2		allow B1 if one term incorrect]
(ii)	Obtain derivative of form $kx^3(3x^4+1)^n$	M1		any constants k and n
	Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$	M1		
	Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$	A1	**3**	or (unsimplified) equiv

Q5, (Jun 2008, Q3)

Attempt use of product rule	**M1**	… + … form
Obtain $2x\ln x + x^2 . \frac{1}{x}$	**A1**	or equiv
Substitute e to obtain 3e for gradient	**A1**	or exact (unsimplified) equiv
Attempt eqn of straight line with numerical gradient	**M1**	allowing approx values
Obtain $y - e^2 = 3e(x - e)$	**A1√**	or equiv; following their gradient provided obtained by diffn attempt; allow approx values
Obtain $y = 3ex - 2e^2$	**A1**	in terms of e now and in requested form
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Q6, (Jan 2010, Q5)

(i)	Obtain derivative of form $kx(x^2+1)^7$	M1		any constant k
	Obtain $16x(x^2+1)^7$	A1		or equiv
	Equate first derivative to 0 and confirm $x=0$ or substitute $x=0$ and verify first derivative zero	M1		AG; allow for deriv of form $kx(x^2+1)^7$
	Refer, in some way, to $x^2+1=0$ having no root	A1	4	or equiv
(ii)	Attempt use of product rule	*M1		obtaining … + … form
	Obtain $16(x^2+1)^7$ + …	A1√		follow their $kx(x^2+1)^7$
	Obtain … + $224x^2(x^2+1)^6$	A1√		follow their $kx(x^2+1)^7$; or unsimplified equiv
	Substitute 0 in attempt at second derivative	M1		dep *M
	Obtain 16	A1	5	from second derivative which is correct at some point
			9	

Q7, (Jun 2010, Q1)

(i)	Attempt use of product rule	M1		producing … + … form
	Obtain $3x^2e^{2x} + 2x^3e^{2x}$	A1	**2**	or equiv
(ii)	Attempt use of chain rule to produce $\frac{kx}{3+2x^2}$ form	M1		any constant k
	Obtain $\frac{4x}{3+2x^2}$	A1	**2**	
(iii)	Attempt use of quotient rule	M1		or equiv; condone u/v confusions
	Obtain $\frac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1} - 2x(2x+1)^{-2}$	A1	**2**	or (unsimplified) equiv

[If $...+c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of $...+c$.]

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Q8, (Jan 2013, Q1)

(i)	Either Attempt use of quotient rule	M1	allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving x; for M1 condone minor errors such as absence of square in denominator, absence of brackets, …
	Obtain $\frac{3(2x+1)-6x}{(2x+1)^2}$ or equiv	A1	give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'
	Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv but A0 for final $\frac{3}{5^2}$
		[3]	
	Or Attempt use of product rule for $3x(2x+1)^{-1}$	M1	allow sign error; condone no use of chain rule
	Obtain $3(2x+1)^{-1}-6x(2x+1)^{-2}$ or equiv	A1	
	Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv
(ii)	Differentiate to obtain form $kx(4x^2+9)^n$	M1	any non-zero constants k and n (including 1 or $\frac{1}{2}$ for n)
	Obtain $4x(4x^2+9)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
	Substitute 2 to obtain $\frac{8}{5}$ or 1.6	A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$
		[3]	

Q9, (Jan 2013, Q4)

(i)	Attempt process involving logarithm to solve $e^{0.021t}=2$	M1	with t the only variable; at least as far as $0.021t=\ln 2$; must be …= 2
	Obtain 33	A1	or greater accuracy; ignore absence of, or wrong, units; final answer $\frac{\ln 2}{0.021}$ is A0
	State (or calculate separately to obtain) 99	B1√	following previous answer; no need to include units
		[3]	
(ii)	Differentiate to obtain $ke^{0.021t}$	M1	where k is any constant not equal to 250
	Obtain $250\times0.021\,e^{0.021t}$	A1	or simplified equiv $5.25e^{0.021t}$
	Substitute to obtain 8.4 or $\frac{42}{5}$	A1	or value rounding to 8.4 with no obvious error
		[3]	

Q10, (Jun 2014, Q1)

Attempt use of product rule to find first derivative	M1	producing form … ± … where one term involves $\ln x$ and the other does not
Obtain $8x\ln x+4x$	A1	or unsimplified equiv
Attempt use of correct product rule to find second derivative	M1	with one term involving $\ln x$
Obtain $8\ln x+12$	A1	or unsimplified equiv
Obtain 28	A1	
	[5]	

Q11, (Jun 2015, Q1)

Attempt use of quotient rule or, after adjustment, product rule	*M1	For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x-8$ in denominator; allow M1 for numerator the wrong way round	For product rule attempt, *M1 for $k_1(3x-8)^{-1}+k_2(5x+4)(3x-8)^{-2}$ form and A1 for correct constants 5 and –3;
Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv	A1	Allow if missing brackets implied by subsequent simplification or calculation	
Substitute 2 to obtain –13 or equiv	A1		
Attempt to find equation of tangent	M1	Dep *M; equation of tangent not normal	
Obtain $y=-13x+19$ or $13x+y-19=0$	A1	Or similarly simplified equiv with 3 non-zero terms	
	[5]		

Q12, (Jun 2016, Q1)

Differentiate to produce form $k_1x(x+2)^m + k_2x^2(x+2)^n$	*M1	For positive integers k_1, k_2, m, n; allow M1 if slip to, for example, $(x+3)$ in both brackets
Obtain $6x(x+2)^6 + 18x^2(x+2)^5$	A1	Or unsimplified equiv
Substitute $x = -1$ to obtain value 12	A1	From correct work only
Attempt equation of tangent (not normal) through point $(-1, 3)$	M1	Dep *M; using non-zero numerical value of gradient; condone slip in use of coordinates
Obtain $y = 12x + 15$	A1 [5]	Answer required in $y = mx + c$ form

Q13, (OCR 4753, Jan 2009, Q2i)

$y = x\cos 2x$	M1	product rule
$\frac{dy}{dx} = -2x\sin 2x + \cos 2x$	B1	d/dx (cos2x) = –2sin2x
	A1	oe cao
	[3]	

Q14, (OCR 4753, Jan 2012, Q1)

$y = x^2 \tan 2x$	M1	product rule	$u \times$ their $v' + v \times$ their u' attempted
	M1	d/du(tan u) = sec^2u soi	M0 if d/dx (tan 2x) =(2) sec^2x
$\Rightarrow$ dy/dx = $2x^2\sec^2 2x + 2x\tan 2x$	A1cao	or $2x^2/\cos^2 2x + 2x\tan 2x$	isw
OR $y = x^2 \frac{\sin 2x}{\cos 2x}$			*see additional notes for complete solution*
$\frac{dy}{dx} = x^2 \frac{\cos 2x.2\cos 2x - \sin 2x(-2\sin 2x)}{\cos^2 2x} + 2x\frac{\sin 2x}{\cos 2x}$	M1	product rule	$u \times$ their $v' + v \times$ their u' attempted
	A1	correct expression	
$= \ldots = 2x^2\sec^2 2x + 2x\tan 2x$	A1cao	or $2x^2/\cos^2 2x + 2x\tan 2x$ (isw)	or $(2x^2 + 2x\sin 2x\cos 2x)/\cos^2 2x$ or $2x^2/\cos^2 2x + 2x\sin 2x / \cos 2x$
OR $y = \frac{x^2 \sin 2x}{\cos 2x}$			*see additional notes for complete solution*
$\frac{dy}{dx} = \frac{\cos 2x(2x\sin 2x + x^2 2\cos 2x) - 2x^2\sin 2x(-\sin 2x)}{\cos^2 2x}$	M1	quotient rule	$(v \times$ their $u' - u \times$ their $v')/v^2$ attempted
	A1	correct expression	
$= \ldots = 2x^2\sec^2 2x + 2x\tan 2x$	A1cao	or $2x^2/\cos^2 2x + 2x\tan 2x$ (isw)	or $(2x^2 + 2x\sin 2x\cos 2x)/\cos^2 2x$ or $2x^2/\cos^2 2x + 2x\sin 2x / \cos 2x$
	[3]		

Q15, (OCR 4753, Jun 2010, Q3)

(i) $y = (1+3x^2)^{1/2}$		
$\Rightarrow$ $dy/dx = \frac{1}{2}(1+3x^2)^{-1/2}.6x$	M1	chain rule
$= 3x(1+3x^2)^{-1/2}$	B1	$\frac{1}{2}u^{-1/2}$
	A1	o.e., but must be '3'
	[3]	
(ii) $y = x(1+3x^2)^{1/2}$		
$\Rightarrow$ $dy/dx = x.\frac{3x}{\sqrt{1+3x^2}} + 1.(1+3x^2)^{1/2}$	M1	product rule
	A1ft	ft their dy/dx from (i)
$= \frac{3x^2+1+3x^2}{\sqrt{1+3x^2}}$	M1	common denominator or factoring $(1+3x^2)^{-1/2}$
$= \frac{1+6x^2}{\sqrt{1+3x^2}}$ *	E1	www
	[4]	