Chain, Product and Quotient Rules Introductory Questions (Sheet 2)

Q1.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dy}{dx} = \frac{(x+1)^2 \times (10x+10) - (5x^2+10x) \times 2(x+1)}{(x+1)^4}$	M1	This mark is given for an attempt to differentiate the expression for y
		A1	This mark is given for correctly differentiating the expression for y
	$\frac{dy}{dx} = \frac{(x+1) \times (10x+10) - (5x^2+10x) \times 2}{(x+1)^3}$	M1	This mark sis given for cancelling the expression through by $(x + 1)$
	$\frac{dy}{dx} = \frac{10}{(x+1)^3}$	A1	This mark is given for a fully correct expression for $\frac{dy}{dx}$
(b)	If $A > 0$ and $n = 1, 3$ then $x < -1$	B1	This mark is given for deducing that $\frac{dy}{dx} < 0 \Rightarrow x < -1$.

Q2.

Question	Scheme	Marks	AOs
	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n=3, A=10, B=1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n=3, A=10, B=1$

Q3.

Question Number	Scheme	Marks
	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

Q4.

Question Number	Scheme	Marks
(a)	$\frac{1}{(x^2+3x+5)}\times ....,= \frac{2x+3}{(x^2+3x+5)}$	M1,A1
		(2)
(b)	Applying $\frac{vu'-uv'}{v^2}$	M1,
	$\frac{x^2\times -\sin x-\cos x\times 2x}{(x^2)^2}=\frac{-x^2\sin x-2x\cos x}{x^4}=\frac{-x\sin x-2\cos x}{x^3}$ oe	A2,1,0
		(3)
		5 Marks

Q5.

Question No	Scheme	Marks
	(a)	
	$\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1
		(4)
	(b)	
	Applying $\frac{vu'-uv'}{v^2}$	
	$\frac{x^3 \times 4cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1 A1
	$= \frac{4xcos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 MARKS)

Q6.

Question Number	Scheme	Marks
	(a)	
	$\frac{dy}{dx} = 2e^{2x}\tan x + e^{2x}\sec^2 x$	M1 A1+A1
	$\frac{dy}{dx} = 0 \Rightarrow 2e^{2x}\tan x + e^{2x}\sec^2 x = 0$	M1
	$2\tan x + 1 + \tan^2 x = 0$	A1
	$(\tan x + 1)^2 = 0$	
	$\tan x = -1$ ∗ cso	A1 (6)
	(b) $\left(\frac{dy}{dx}\right)_0 = 1$	M1
	Equation of tangent at $(0, 0)$ is $y = x$	A1 (2)
		[8]

Q7.

Question Number	Scheme		Marks
Q (i)(a)	$y = x^2\cos 3x$		
	Apply product rule: $\begin{cases} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{cases}$		
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2\sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.	A1
			(3)
(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$	$\ln(x^2+1) \to \frac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \to \frac{2x}{x^2+1}$	A1
	Apply quotient rule: $\begin{cases} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{cases}$		
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
(ii)	$y=\sqrt{4x+1},\ x>-\frac{1}{4}$		
	At P, $y=\sqrt{4(2)+1}=\underline{\sqrt{9}}=\underline{3}$	At P, $y=\underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\frac{dy}{dx}=\underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\frac{dy}{dx}=\frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{dy}{dx}=\frac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x=2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence $m(\mathbf{T})=\frac{2}{3}$		
	Either $\mathbf{T}$: $y-3=\frac{2}{3}(x-2)$; or $y=\frac{2}{3}x+c$ and $3=\frac{2}{3}(2)+c \Rightarrow c=3-\frac{4}{3}=\frac{5}{3}$;	$y-y_1=m(x-2)$ or $y-y_1=m(x-\text{their stated }x)$ with 'their TANGENT gradient' and their y_1; or uses $y=mx+c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either $\mathbf{T}$: $3y-9=2(x-2)$;		
	$\mathbf{T}$: $3y-9=2x-4$		
	$\mathbf{T}$: $\underline{2x-3y+5=0}$	$\underline{2x-3y+5=0}$ Tangent must be stated in the form $ax+by+c=0$, where a, b and c are integers.	A1
			(6)
	or $\mathbf{T}$: $y=\frac{2}{3}x+\frac{5}{3}$		
	$\mathbf{T}$: $3y=2x+5$		
	$\mathbf{T}$: $\underline{2x-3y+5=0}$		
			[13]

Q8.

Question Number	Scheme	Marks
	At P, $y = \underline{3}$	B1
	$\frac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$	M1<u>A1</u>
	$\frac{dy}{dx} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$m(N) = \frac{-1}{-18}$ or $\frac{1}{18}$	M1
	N: $y - 3 = \frac{1}{18}(x-2)$	M1
	N: $\underline{x - 18y + 52 = 0}$	A1
		[7]
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$; 3rd M1: Uses $m(N) = -\frac{1}{\text{their } m(T)}$. 4th M1: $y - y_1 = m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Q9.

Question	Scheme	Marks	AOs
	$y = \frac{x-4}{2+\sqrt{x}} \Rightarrow \frac{dy}{dx} = \frac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$= \frac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2} = \frac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2} = \frac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$= \frac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2} = \frac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2} = \frac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
		(4 marks)	
	Notes		

Q10.

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx}=\sqrt{3}e^{\sqrt{3}x}\sin 3x+3e^{\sqrt{3}x}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $\quad e^{\sqrt{3}x}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\tan 3x=-\sqrt{3}$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)
	(b) At $x=0$ $\quad \frac{dy}{dx}=3$	B1
	Equation of normal is $-\frac{1}{3}=\frac{y-0}{x-0}$ or any equivalent $y=-\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

(a) M1 Applies the product rule vu'+uv' to $e^{\sqrt{3}x}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=….,u'=….,v=…..,v'=….followed by their vu'+uv') only accept answers of the form $\frac{dy}{dx}=Ae^{\sqrt{3}x}\sin 3x+e^{\sqrt{3}x}\times\pm B\cos 3x$

A1 Correct expression for $\frac{dy}{dx}=\sqrt{3}e^{\sqrt{3}x}\sin 3x+3e^{\sqrt{3}x}\cos 3x$

M1 Sets **their** $\frac{dy}{dx}=0$, factorises out or divides by $e^{\sqrt{3}x}$ producing an equation in $\sin 3x$ and $\cos 3x$

A1 Achieves either $\tan 3x=-\sqrt{3}$ or $\tan 3x=-\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x=\frac{5\pi}{3}\Rightarrow x=\frac{5\pi}{9}$ or $3x=\frac{-\pi}{3}\Rightarrow x=\frac{-\pi}{9}$ but not $x=\arctan(\frac{-\sqrt{3}}{3})$

A1 CSO $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with (0, 0).

Eg $-\frac{1}{their\ 'm'}=\frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y=-\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3}=\frac{y-0}{x-0}$.