Differentiation (Chain, Product and Quotient Rules) Harder Exam Questions (From OCR 4723)

Q1, (Jun 2005, Q6)

(a) Find the exact value of the x-coordinate of the stationary point of the curve $y = x \ln x$. [4]

(b) The equation of a curve is $y = \frac{4x + c}{4x - c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]

Q2, (Jun 2007, Q8i,ii)

(i) Given that $y = \frac{4 \ln x - 3}{4 \ln x + 3}$, show that $\frac{dy}{dx} = \frac{24}{x(4 \ln x + 3)^2}$. [3]

(ii) Find the exact value of the gradient of the curve $y = \frac{4 \ln x - 3}{4 \ln x + 3}$ at the point where it crosses the x-axis. [4]

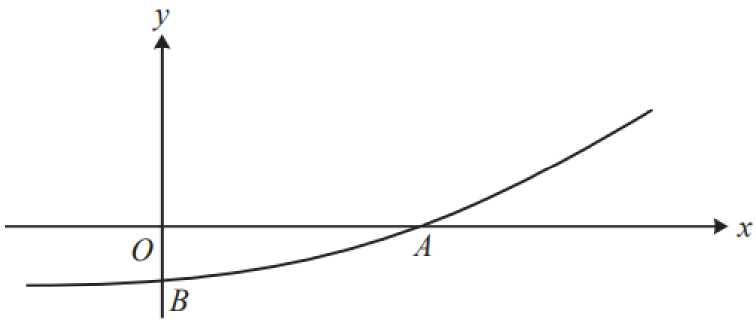
Q3, Jan 2008, Q7)

A curve has equation $y = \frac{xe^{2x}}{x + k}$, where k is a non-zero constant.

(i) Differentiate xe^{2x}, and show that $\frac{dy}{dx} = \frac{e^{2x}(2x^2 + 2kx + k)}{(x + k)^2}$. [5]

(ii) Given that the curve has exactly one stationary point, find the value of k, and determine the exact coordinates of the stationary point. [5]

Q4, (Jan 2013, Q7)



The diagram shows the curve with equation

$$x = (y + 4)\ln(2y + 3).$$

The curve crosses the x-axis at A and the y-axis at B.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. **[3]**

(ii) Find the gradient of the curve at each of the points A and B, giving each answer correct to 2 decimal places. **[5]**

Q5, (Jan 2011, Q9)

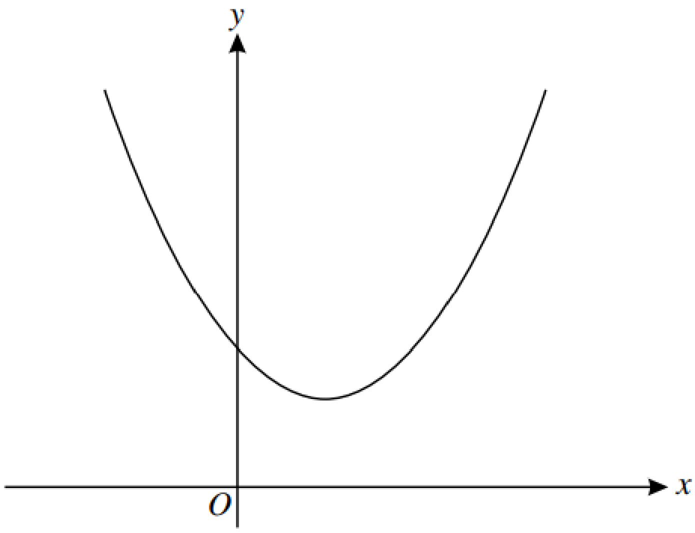
(i) The function f is defined for all real values of x by

$$f(x) = e^{2x} - 3e^{-2x}.$$

(a) Show that $f'(x) > 0$ for all x. **[3]**

(b) Show that the set of values of x for which $f''(x) > 0$ is the same as the set of values of x for which $f(x) > 0$, and state what this set of values is. **[5]**

(ii)



The function g is defined for all real values of x by

$$g(x) = e^{2x} + ke^{-2x},$$

where k is a constant greater than 1. The graph of $y = g(x)$ is shown above. Find the range of g, giving your answer in simplified form. **[5]**

Q6, (Jun 2016, Q6)

The curves C_1 and C_2 have equations

$$y = \ln(4x - 7) + 18 \quad \text{and} \quad y = a(x^2 + b)^{\frac{1}{2}}$$

respectively, where a and b are positive constants. The point P lies on both curves and has x-coordinate 2. It is given that the gradient of C_1 at P is equal to the gradient of C_2 at P. Find the values of a and b. **[8]**

Q7, (Jun 2017, Q9)

(a) The equation of a curve has the form $y = \dfrac{px+q}{x^2+3}$. Show that the curve has two distinct stationary points for all non-zero values of the constants p and q. **[4]**

(b) The equation of a curve has the form $y = e^{x^2}(ax^2+b)$, where a and b are non-zero constants. It is given that $\dfrac{d^2y}{dx^2}$ can be expressed in the form $e^{x^2}(cx^4+d)$, where c and d are non-zero constants. Prove that $5a+2b=0$. **[5]**

Q8, (OCR 4753, Jan 2010, Q8i,ii)

Fig. 8 shows part of the curve $y = x\cos 3x$.

The curve crosses the x-axis at O, P and Q.

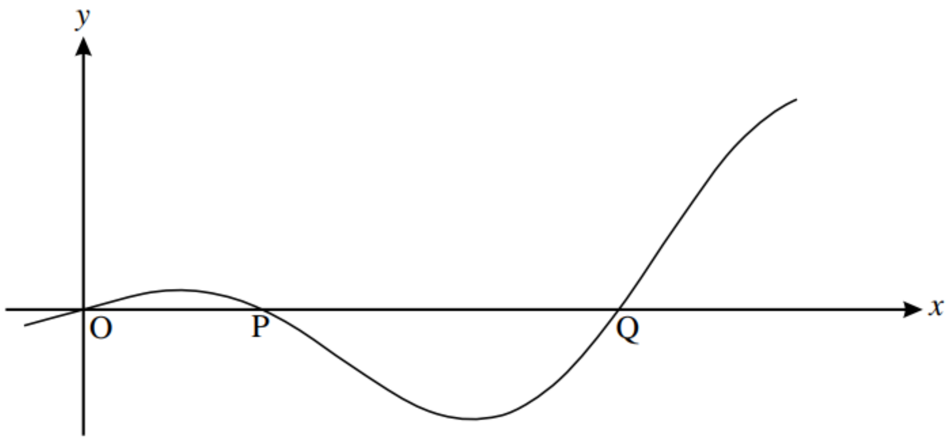


Fig. 8

(i) Find the exact coordinates of P and Q. **[4]**

(ii) Find the exact gradient of the curve at the point P.

Show also that the turning points of the curve occur when $x\tan 3x = \frac{1}{3}$. **[7]**
