Differentiation (Chain, Product and Quotient Rules) Harder Exam Questions MS (From OCR 4723)

Q1, (Jun 2005, Q6)

(a)	Attempt use of product rule	*M1	
	Obtain $\ln x + 1$	A1	[or unsimplified equiv]
	Equate attempt at first derivative to zero and obtain value involving e	M1	[dependent on *M]
	Obtain e^{-1}	A1	4 [or exact equiv]
(b)	Attempt use of quotient rule	M1	[or equiv using product rule or …]
	Obtain $\dfrac{(4x-c)4-4(4x+c)}{(4x-c)^2}$	A1	[or equiv]
	Show that first derivative cannot be zero	A1	3 [AG; derivative must be correct]

Q2, (Jun 2007, Q8i,ii)

(i)	Attempt use of quotient rule	M1		allow for numerator 'wrong way round'; or equiv
	Obtain $\dfrac{(4\ln x+3)\frac{4}{x}-(4\ln x-3)\frac{4}{x}}{(4\ln x+3)^2}$	A1		or equiv
	Confirm $\dfrac{24}{x(4\ln x+3)^2}$	A1	3	AG; necessary detail required
(ii)	Identify $\ln x = \frac{3}{4}$	B1		or equiv
	State or imply $x = e^{\frac{3}{4}}$	B1		
	Substitute e^k completely in expression for derivative	M1		and deal with $\ln e^k$ term
	Obtain $\frac{2}{3}e^{-\frac{3}{4}}$	A1	4	or exact (single term) equiv

Q3, Jan 2008, Q7)

(i)	Attempt use of product rule for xe^{2x}	M1		obtaining … + …
	Obtain $e^{2x} + 2xe^{2x}$	A1		or equiv; maybe within QR attempt
	Attempt use of quotient rule	M1		with or without product rule
	Obtain unsimplified $\dfrac{(x+k)(e^{2x}+2xe^{2x})-xe^{2x}}{(x+k)^2}$	A1		
	Obtain $\dfrac{e^{2x}(2x^2+2kx+k)}{(x+k)^2}$	A1	5	AG; necessary detail required
(ii)	Attempt use of discriminant	M1		or equiv
	Obtain $4k^2-8k=0$ or equiv and hence $k=2$	A1		
	Attempt solution of $2x^2+2kx+k=0$	M1		using their numerical value of k or solving in terms of k using correct formula
	Obtain $x=-1$	A1		
	Obtain $-e^{-2}$	A1	5	or exact equiv

Q4, (Jan 2013, Q7)

(i)	Attempt use of product rule	M1	to produce expression of form (something non-zero) $\ln(2y+3)$ + $\frac{\text{linear in } y}{\text{linear in } y}$; ignore what they call their derivative
	Obtain $\ln(2y+3)$ …	A1	with brackets included
	Obtain … + $\frac{2(y+4)}{2y+3}$	A1	with brackets included as necessary
		[3]	
(ii)	Substitute $y=0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
	Obtain 0.27 for gradient at *A*	A1	or greater accuracy 0.26558…; beware of 'correct' answer coming from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer in part (i)
	Attempt to find value of *y* for which $x=0$	M1	allowing process leading only to $y=-4$
	Substitute $y=-1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
	Obtain 0.17 or $\frac{1}{6}$ for gradient at *B*	A1	or greater accuracy 0.16666…; value following from correct working
		[5]	

Q5, (Jan 2011, Q9)

(i)	**(a)** Differentiate to obtain $k_1e^{2x}+k_2e^{-2x}$	M1		any constants k_1 and k_2 but derivative must be different from f(*x*); condone presence of + *c*
	Obtain $2e^{2x}+6e^{-2x}$	A1		or unsimplified equiv; no + *c* now
	Refer to $e^{2x}>0$ and $e^{-2x}>0$ or to more general comment about exponential functions	A1	**3**	or equiv (which might be sketch of $y=f(x)$ with comment that gradient is positive or might be sketch of $y=f'(x)$ with comment that $y>0$; AG
	(b) Differentiate to obtain $k_3e^{2x}+k_4e^{-2x}$	M1		any constants k_3 and k_4 but second derivative must be different from their first derivative; condone presence of + *c*
	Obtain $4e^{2x}-12e^{-2x}$	A1		or unsimplified equiv; no + *c* now
	Attempt solution of $f''(x)>0$ or of $f(x)>0$ or of corresponding eqn	M1		at least as far as term involving e^{4x} or e^{-4x}
	Obtain $x>\frac{1}{4}\ln 3$	A1		
	Confirm both give same result	B1	**5**	AG; necessary detail needed; either by solving the other or by observing that same inequality involved (just noting that $f''(x)=4f(x)$ is sufficient)
(ii)	Differentiate to obtain $2e^{2x}-2ke^{-2x}$	B1		or unsimplified equiv
	Attempt to find *x*-coordinate of stationary pt	M1		equating to 0 and reaching $e^{4x}=$ ... or equiv
	Obtain $e^{4x}=k$ and hence $\frac{1}{4}\ln k$ or equiv	A1		or equiv such as $e^{2x}=\sqrt{k}$
	Substitute and attempt simplification	M1		using valid processes but allow if only limited progress [note that question can be successfully concluded (without actually finding *x*) by substitution of $e^{2x}=\sqrt{k}$ and $e^{-2x}=\frac{1}{\sqrt{k}}$]
	Obtain $g(x)\geq 2\sqrt{k}$ or $y\geq 2\sqrt{k}$	A1	**5**	or similarly simplified equiv with ≥ not >
			13	

Q6, (Jun 2016, Q6)

State, at some stage, $a(4+b)^{\frac{1}{2}}=18$	B1	
Obtain derivative $\frac{4}{4x-7}$ for C_1	B1	
Obtain derivative $kx(x^2+b)^{-\frac{1}{2}}$ for C_2	M1	Any non-zero constant k
Obtain correct $ax(x^2+b)^{-\frac{1}{2}}$	A1	
Equate derivatives with $x=2$	M1	
Attempt values of a and b from two equations involving a and $(4+b)^{\frac{1}{2}}$	M1	Using correct process Correct equations are $a(4+b)^{\frac{1}{2}}=18$ and $2a(4+b)^{-\frac{1}{2}}=4$
Obtain $a=6$	A1	
Obtain $b=5$	A1	
	[8]	

Q7, (Jun 2017, Q9)

a	Differentiate using quotient rule or equiv	M1	With negative sign in numerator, with $(x^2+3)^2$ in denominator and at least one of the two terms in the numerator correct
	Obtain $\frac{p(x^2+3)-2x(px+q)}{(x^2+3)^2}$ or equiv	A1	
	Equate derivative to zero and attempt discriminant	M1	Provided equation is a 3-term quadratic with p and q present
	Obtain $4q^2+12p^2$ and observe it is positive	A1	With at least one reference to squared value being positive
		[4]	
b	Differentiate to obtain form $e^{x^2}(px^3+qx)$	M1	
	Obtain $\frac{dy}{dx}=2xe^{x^2}(ax^2+b)+2axe^{x^2}$	A1	Or equiv
	Obtain $\frac{d^2y}{dx^2}=e^{x^2}(4ax^4+10ax^2+4bx^2+2a+2b)$	A1	Or equiv
	Equate coefficient of $x^2e^{x^2}$ to zero	M1	Provided second derivative involves $e^{x^2}x^4$, $e^{x^2}x^2$ and e^{x^2} terms and no others
	Confirm $5a+2b=0$	A1	AG – necessary detail needed
		[	

Q8, (OCR 4753, Jan 2010, Q8i,ii)

(i) At P, $x\cos 3x=0$		
$\Rightarrow$ $\cos 3x=0$	M1	or verification
$\Rightarrow$ $3x=\pi/2,\ 3\pi/2$	M1	$3x=\pi/2,\ (3\pi/2\ldots)$
$\Rightarrow$ $x=\pi/6,\ \pi/2$	A1 A1	dep both Ms condone degrees here
So P is $(\pi/6, 0)$ and Q is $(\pi/2, 0)$	[4]	
(ii) $\frac{dy}{dx}=-3x\sin 3x+\cos 3x$	M1 B1 A1	Product rule d/dx $(\cos 3x)=-3\sin 3x$ cao (so for $dy/dx=-3x\sin 3x$ allow B1) mark final answer
At P, $\frac{dy}{dx}=-\frac{\pi}{2}\sin\frac{\pi}{2}+\cos\frac{\pi}{2}=-\frac{\pi}{2}$	M1 A1cao	substituting their $-\pi/6$ (must be rads) $-\pi/2$
At TPs $\frac{dy}{dx}=-3x\sin 3x+\cos 3x=0$	M1	$dy/dx=0$ and $\sin 3x/\cos 3x=\tan 3x$ used
$\Rightarrow$ $\cos 3x=3x\sin 3x$		
$\Rightarrow$ $1=3x\sin 3x/\cos 3x=3x\tan 3x$		
$\Rightarrow$ $x\tan 3x=1/3$ *	E1	www
	[7]	