

Differential Equations Exam Questions MS (From OCR 4724)

Q1, (Jan 2006, Q8)

(i)	$\int (y-3) dy = \int (2-x) dx$ or equiv	M1	For separation & integration of both sides
	$\frac{1}{2}y^2 - 3y = 2x - \frac{1}{2}x^2$	A1	or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2$
	For an arbitrary const on one/both sides	*B1	} (or + M2 for equiv statement using limits)
	Substituting $(x, y) = (5, 4)$ or $(4, 5)$ & finding 'c' dep*M1		
	$\frac{1}{2}y^2 - 3y = -\frac{1}{2}x^2 + 2x - \frac{3}{2}$	AEF ISW A1	5 or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2 + 5$ AEF

(ii)	Attempt to clear fracts (if nec) & compl square	M1	3 For all 3; SR: A1 for 1 or 2 correct
	$a = 2, b = 3, k = 10$	A2	

(iii)	Circle clearly indicated in a sketch	B1	3 $\sqrt{\text{provided } k > 0}$
	Centre $(2, 3)$ or their (a, b)	B1 \checkmark	
	Radius $\sqrt{10}$ or their \sqrt{k}	B1 \checkmark	

Q2, (Jan 2007, Q9)

(i) Separate variables as $\int \sec^2 y dy = 2 \int \cos^2 2x dx$	M1	seen or implied	
LHS = $\tan y$	A1		
RHS; attempt to change to double angle	M1		
Correctly shown as $1 + \cos 4x$	A1		
$\int \cos 4x dx = \frac{1}{4} \sin 4x$	A1		
Completely correct equation (other than +c)	A1		
+c on either side	A1		7 <u>not</u> on both sides unless c_1 and c_2
(ii) Use boundary condition	M1		provided a sensible outcome would ensue
c (on RHS) = 1	A1		or $c_2 - c_1 = 1$; not fortuitously obtained
Substitute $x = \frac{1}{6}\pi$ into their eqn, produce $y = 1.05$	A1		3 or 4.19 or 7.33 etc. Radians only

Q3, (Jun 2007, Q8)

(i) Sep variables eg $\int \frac{1}{6-h} (dh) = \int \frac{1}{20} (dt)$	*M1		s.o.i. Or $\frac{dt}{dh} = \frac{20}{6-h} \rightarrow$ M1
LHS = $-\ln(6-h)$	A1		& then $t = -20 \ln(6-h) (+c) \rightarrow$ A1+A1
RHS = $\frac{1}{20}t (+c)$	A1		
Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
Correct value of their c = $-(20)\ln 5$ WWW	A1		or $(20)\ln 5$ if on LHS
Produce $t = 20 \ln \frac{5}{6-h}$ WWW AG	A1	6	Must see $\ln 5 - \ln(6-h)$
(ii) When $h = 2, t = 20 \ln \frac{5}{4} = 4.46(2871)$	B1	1	Accept 4.5, $4\frac{1}{2}$
(iii) Solve $10 = 20 \ln \frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$	M1		or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$ -way stage
$h = 2.97(2.9673467\dots)$	A1	2	$6 - 5e^{-0.5}$ or $6 - e^{1.109}$
[In (ii),(iii) accept non-decimal (exact) answers but - 1 once.] Accept truncated values in (ii),(iii).			
(iv) Any indication of (approximately) 6 (m)	B1	1	

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Q4, (Jan 2008, Q8)

(i) $\frac{dx}{dt}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen	M1		k non-numerical; i.e. 1 side correct
$\frac{dx}{dt} = -kx^{\frac{1}{2}}$ or $\frac{dx}{dt} = kx^{\frac{1}{2}}$	A1	2	i.e. both sides correct

(ii) Separate variables or invert, + attempt to integrate	*M1		Based <u>only</u> on above eqns or $\frac{dx}{dt} = x^{\frac{1}{2}}, -x^{\frac{1}{2}}$
Correct result for their equation after integration	A1		Other than omission of 'c'
Subst $(t,x) = (0,2)$ into eqn containing k &/or c	dep*M1		or substitute (5,1)
Subst $(t,x) = (5,1)$ into eqn containing k & c	dep*M1		or substitute (0,2)
Subst $x = 0.5$ into eqn with their k & c subst	dep*M1		
$t = 8.5(8.5355339)$	A1	6	[1 d.p. requested in question]

Q5, (Jan 2009, Q9)

- (i) $\frac{d\theta}{dt} = \dots$ B1
 $k(160 - \theta)$ B1 (2) The 2 @ 'B1' are indep
- (ii) Separate variables with $(160 - \theta)$ in denom; or invert *M1 $\int \frac{1}{160 - \theta} d\theta = \int k, \frac{1}{k}, 1 dt$
 Indication that LHS = $\ln f(\theta)$ A1 If wrong ln, final 3@A = 0
 RHS = kt or $\frac{1}{k}t$ or t (+ c) A1
 Subst. $t = 0, \theta = 20$ into equation containing 'c' dep* M1
 Subst $t = 5, \theta = 65$ into equation containing 'c' & 'k' dep* M1
 $c = -\ln 140$ (-4.94) ISW A1
 $k = \frac{1}{5} \ln \frac{140}{95}$ (≈ 0.077 or 0.078) ISW A1
 Using their 'c' & 'k', subst $t = 10$ & evaluate θ dep* M1
 $\theta = 96(95.535714)$ ($95 \frac{15}{28}$) A1 (9)

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Q6, (Jan 2011, Q9)

- (i) Attempt to sep variables in the form $\int \frac{P}{(x-8)^{2/3}} dx = \int q dt$ M1 Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{2/3}}$; p, q, r const
 $\int \frac{1}{(x-8)^{2/3}} dx = k(x-8)^{2/3}$ A1 k const
 All correct (+ c) A1
 For equation containing 'c'; substitute $t = 0, x = 72$ M1 M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
 Correct corresponding value of c from correct eqn A1
 Subst their c & $x = 35$ back into eqn M1
 $t = \frac{21}{8}$ or $2.63 / 2.625$ [C.A.O] A1 7 A2: $t = \frac{21}{8}$ or $2.63 / 2.625$ WWW
 (ii) State/imply in some way that $x = 8$ when flow stops B1
 Substitute $x = 8$ back into equation containing numeric 'c' M1
 $t = 6$ A1 3

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Q7, (Jun 2014, Q10)

(i)	$\frac{dV}{dt} = \pm 0.01$ <p>by similar triangles, $\frac{h}{4.5} = \frac{r}{3}$</p> $\frac{dV}{dh} = \frac{4}{9}\pi h^2 \text{ oe}$ $\frac{dh}{dt} = \pm 0.01 \times \text{their } \frac{dh}{dV} \text{ oe}$ $-0.01 = \left(\frac{4}{9}\pi h^2\right) \times \frac{dh}{dt}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>may be implied by $r = \frac{2h}{3}$ oe</p> <p>use of Chain rule</p> <p>completion to given result www</p>	<p>may follow from incorrect differentiation: expressions must be a function of either r or h or both</p> $h^2 \frac{dh}{dt} = \frac{-0.09}{4\pi} = \frac{-9}{400\pi}$
(ii)	$\int h^2 dh = \int \frac{-9}{400\pi} dt \text{ oe soi}$ $\frac{h^3}{3} = \frac{-9}{400\pi} t(+c)$ <p>substitution of $t = 0$ and $h = 4.5$ in their expression following integration</p> $h = \sqrt[3]{\frac{729}{8} - \frac{27t}{400\pi}} \text{ oe isw}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>separation of variables</p> <p>expression must include c and powers must be correct on each side</p> <p>allow -0.0215 or $-0.02148591\dots$ r.o.t to 4 sf or more and similarly 91.125</p>	<p>if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;</p> <p>$91.125 = \sqrt[3]{729/8}$</p>
(iii)	<p>set $h = 0$ and solve to obtain positive t</p> <p>71 minutes cao</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>or $(t =) \frac{1}{3}\pi \times 3^2 \times 4.5 \div 0.01 (= 1350\pi)$</p>	<p>NB $1350\pi = 4241.150082\dots$</p>

Q8, (Jun 2015, Q8)

<p>(i)</p>	$\frac{dP}{dt} = \frac{k}{P}$ <p>$k = 100$ from $\frac{dP}{dt} = \frac{k}{P}$</p> $\int P dP = \int (\text{their } k) dt$ $\frac{P^2}{2} = kt + c$ <p>substitution of $t = 0$ and $P = 100$</p> $P = \sqrt{10000 + 200t}$ or $10\sqrt{100 + 2t}$ or $P = \sqrt{200(50 + t)}$ isw cao	<p>B1</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>[6]</p>	<p>or $\frac{dP}{dt} = \frac{1}{kP}$</p> <p>or $k = 0.01$ from $\frac{dP}{dt} = \frac{1}{kP}$</p> <p>allow $k = 1$</p> <p>or $t = \frac{P^2}{2k} + d$</p> <p>may follow incorrect algebraic manipulation, but equation must include c (or d)</p>	<p>k should be unspecified at this stage</p> <p>may be seen later</p> <p>allow omission of \int and recovery of omission of one operator for M1*A1</p> <p>if M0, SC2 for $\ln P = kt + c$ thereafter only M1 may be earned</p> <p>NB $c = 5000$ or $d = -50$</p> <p>allow recovery from eg use of x for P throughout, but withhold final A1 for eg $x = \sqrt{10000 + 200t}$</p>
<p>(ii)</p>	<p>$t = 8, P = 107.7$ or 108 so model was a good fit in 2008 oe</p> <p>$t = 13, P = 112(.2)$, so model was not appropriate in 2013 oe</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>or $t = 8.3(2)$ when $P = 108$ + comment</p> <p>or $t = 31.9(2)$ or 32 when $P = 128$ + comment</p> <p>comments may be in same sentence, but both values must be referenced</p>	<p>value of P or t must be found and correct comment made in each case; comments may be in same sentence.</p> <p>if B0B0, SC1 for both values found no FT marks available</p> <p>comments on trends, extrapolation etc do not score</p> <p>just ticks / crosses etc do not score</p>

Q9, (Jun 2016, Q10)

(i)

$$\frac{A}{(x+4)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$[16 + 5x - 2x^2] = A(x+1)^2 + B(x+1)(x+4) + C(x+4)$$

$$A = -4$$

$$C = 3$$

$$B = 2 \text{ isw}$$

B1

M1

A1

A1

A1

[5]

NB

$$36 = -9A$$

$$9 = 3C$$

$$\begin{aligned} -2 &= A + B, 5 = 2A + 5B + C \\ 16 &= A + 4B + 4C \end{aligned}$$

NB $\frac{-4}{(x+4)} + \frac{2}{(x+1)} + \frac{3}{(x+1)^2}$

may be awarded later

allow sign errors only

if **B0M0**, allow **SC3** for

$$\frac{2x+5}{(x+1)^2} - \frac{4}{x+4}$$

(ii)

$$\int \frac{dy}{y} = \int \frac{16 + 5x - 2x^2}{(x+1)^2(x+4)} dx$$

$$\frac{3}{(x+1)^2} + \frac{2}{(x+1)} - \frac{4}{(x+4)} \text{ seen in RHS, may be embedded}$$

$$\frac{-3}{x+1} + 2\ln(x+1) - 4\ln(x+4) + c$$

$$\ln\left(\frac{1}{256}\right) = -3 + 2\ln 1 - 4\ln 4 + c$$

$$c = 3 \text{ cao}$$

$$\ln y = \frac{-3}{2+1} + 2\ln(2+1) - 4\ln(2+4) + 3$$

$$y = \frac{e^2}{144} \text{ oe}$$

B1

separation of variables

allow omission of integral signs;
allow omission of dy or dx but not both

M1*

FT their partial fractions if two or three terms; ignore LHS

may be implied by correct integration of two of their terms

A1FT

FT their non-zero 3, 2 and 4; allow recovery from $x + 1^2$ in denominator;
if brackets in log terms omitted, allow **A1** if recovery seen in substitution

allow omission of + c here

M1*dep

substitution of $x = 0$ and $y = \frac{1}{256}$;
allow if error in manipulation following integration;

+ c must be included and LHS must be correctly obtained

A1

or $A = e^{-3}$ from $y = Ae^{\frac{-3}{x+1}} \frac{(x+1)^2}{(x+4)^4}$

M1*dep

substitution of $x = 2$;
dependent on award of previous **M1M1** and numerical value found for c

allow **M1** if substitution follows incorrect manipulation eg to find expression for y

[7]

Q10, (Jun 2017, Q7)

<p>i</p>	<p>$\ln A - \ln(250 - A) = kt (+ c)$</p> <p>valid substitution of $t = 0$ and $A = 10$ to find c</p> <p>$c = -\ln 24$ oe</p> <p>constructive log step</p> <p>taking exponentials correctly of both sides; FT their rearrangement and/or <i>their</i> numerical c</p> $[A] = \frac{250e^{kt}}{24 + e^{kt}} \text{ oe}$	<p>M1* allow sign error</p> <p>M1dep* NB $\ln 10 - \ln 240 = 0 + c$</p> <p>A1 allow to 3 sf or more</p> <p>A1 eg $\ln\left(\frac{A}{250 - A}\right) = kt - \ln 24$ oe</p> <p>M1dep* eg $\left(\frac{24A}{250 - A}\right) = e^{kt}$</p> <p>A1</p>	<p>-3.17805383</p> <p>or $\ln\left(\frac{A}{250 - A}\right) = kt - 3.178$</p> <p>or $\left(\frac{A}{250 - A}\right) = e^{kt - 3.178}$</p>
.....			
	<p><i>Alternatively</i></p> <p>$\ln A - \ln(250 - A) = kt (+ c)$</p> <p>constructive log step, may be awarded after taking exponentials</p> <p>taking exponentials correctly of both sides; FT their rearrangement</p> <p>valid substitution of $t = 0$ and $A = 10$ to find c</p> $\frac{A}{250 - A} = e^{kt - \ln 24} \text{ oe}$ $[A] = \frac{250e^{kt}}{24 + e^{kt}} \text{ oe}$	<p>M1* allow sign error</p> <p>A1 eg $\ln\left(\frac{A}{250 - A}\right) = kt + c$</p> <p>M1dep* eg $\frac{A}{250 - A} = e^{kt+c}$</p> <p>M1dep* eg $\frac{10}{250 - 10} = e^{0+c}$</p> <p>A1</p> <p>A1</p> <p>[6]</p>	

ii	$k = 0.05$	B1 [1]		
iii	$A = 250 \text{ [m}^2\text{]}$	B1 [1]	ignore commentary	

Q11, (Jun 2018, Q9)

(i)	$\frac{dV}{dt} = k e^{-t} \text{ soi}$ <p>substitution of $\frac{dV}{dt} = 10, t = 2$</p> $V = -k e^{-t} + c \text{ oe}$ <p>substitution of $t = 0, V = 0$ in their $V = -k e^{-t} + c$</p> $V = 10e^2(1 - e^{-t}) \text{ AG}$ <p>$x = 3$ gives $V = 63$ soi</p> <p>t obtained from $63 = 10e^2(1 - e^{-t})$</p> <p>$t = 1.9$ to 1.915</p>	<p>B1</p> <p>M1</p> <p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>B0 if x used as constant instead of k; M marks are still available</p> <p>NB $10 = k e^{-2}$ not from V</p> <p>from integration; allow omission of $+c$ here; allow eg $V = l e^{-t} + c$</p> <p>must see $+c$ here; may be awarded before k found</p> <p>correct completion to given result; A0 if $73.89\dots$ used in working</p> <p>NB $1.914688\dots$ unsupported to 3 or more sf implies B1M1A1</p>	<p><i>alternatively, from connected rates of change</i></p> $(x+1)^3 = -k e^{-t} + c \text{ from } \int k e^{-t} dt = \int 3(x+1)^2 dx$ <p>substitution of $t = 0, x = 0$ (to obtain $c = 1 + k$)</p>
(ii)	<p>as t becomes large, e^{-t} becomes very small soi</p> <p>x obtained from $10e^2 = (x+1)^3 - 1$</p> <p>$x = 3.22$ cao</p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>need not be explicitly stated</p> <p>allow B3 for 3.22 unsupported</p>	<p>may be implied by $73.890\dots$ to 3 or more sf</p> <p>NB 3.215111183 unsupported to 3 or more sf justifies award of M1M1 if working not seen</p>
		[8]		
		[3]		