Differential Equations (Sheet 2) Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$ so $2 \equiv A(2+y) + B(2-y)$	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$, Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	M1
	giving $\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}$	A1 cao (3)
(b)	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$	
	$\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) \quad + (c)$	B1 M1 A1 ft
	$y=0,\ x=\frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos(\frac{\pi}{3})}\right) + c$	M1
	$\{0 = \ln 2 + c \Rightarrow c = -\ln 2\}$	
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$	
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$	
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	M1
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$	
	Hence, $\sec^2 x = \frac{8+4y}{2-y}$	A1 (8)
		(11 marks)

Q2.

Question	Scheme	Marks	AOs
(a)	$\frac{dH}{dt} = \frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}\,dH = \int \frac{\cos 0.25t}{40}\,dt$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t\,(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t = 0, H = 5 \Rightarrow c = \ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10}\sin 0.25t \Rightarrow H = 5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1} = 5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

Q3.

Question Number	Scheme		Marks
	$\frac{d\theta}{dt} = \lambda(120 - \theta)$, $\theta \leqslant 100$		
(a)	$\int \frac{1}{120-\theta} d\theta = \int \lambda \, dt$ or $\int \frac{1}{\lambda(120-\theta)} d\theta = \int dt$		B1
	$-\ln(120-\theta); = \lambda t + c$ or $-\frac{1}{\lambda}\ln(120-\theta); = t + c$	See notes	M1 A1; M1 A1
	$\{t = 0, \theta = 20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either... $-\lambda t = \ln(120-\theta) - \ln 100$; $-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$; $e^{-\lambda t} = \frac{120-\theta}{100}$ *or...* $\lambda t = \ln 100 - \ln(120-\theta)$; $\lambda t = \ln\left(\frac{100}{120-\theta}\right)$; $e^{\lambda t} = \frac{100}{120-\theta}$		dddM1
	$100e^{-\lambda t} = 120 - \theta$ or $(120-\theta)e^{\lambda t} = 100 \Rightarrow 120 - \theta = 100e^{-\lambda t}$; leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01, \theta = 100 \Rightarrow\}\ 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120 - 100 \Rightarrow -0.01t = \ln\left(\frac{120-100}{100}\right)$; $t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = \ldots$ and $t = A\ln B$, where $B > 0$	dM1
	$\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379\ldots = 161$ (s) (nearest second)	awrt 161	A1
			[3]
			11

Q4.

Question Number	Scheme		Marks
(a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$(V = 4000h \Rightarrow)\ \frac{dV}{dh} = 4000$	$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 AG
			[3]
(b)	When $h = 25$ water *leaks out such that* $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 AG
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 AG
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	*Separates the variables* with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$	Correct proof	A1 AG
			[2]

Question Number	Scheme		Marks
(d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh$ with substitution $h=(20-x)^2$		
	$\frac{dh}{dx} = 2(20-x)(-1)$ or $\frac{dh}{dx} = -2(20-x)$	Correct $\frac{dh}{dx}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,dh = \int \frac{50}{x}.-2(20-x)\,dx$	$\pm\lambda\int\frac{20-x}{x}\,dx$ or $\pm\lambda\int\frac{20-x}{20-(20-x)}\,dx$ where λ is a constant	M1
	$=100\int\frac{x-20}{x}\,dx$		
	$=100\int\left(1-\frac{20}{x}\right)dx$		
	$=100(x-20\ln x)\ (+c)$	$\pm\alpha x \pm \beta\ln x\,;\ \alpha,\beta\neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh = \left[100x-2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$=(1000-2000\ln 10)-(2000-2000\ln 20)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$=2000\ln 20-2000\ln 10-1000$	Combining logs to give...	
	$=2000\ln 2-1000$	$2000\ln 2-1000$ or $-2000\ln\left(\frac{1}{2}\right)-1000$	A1 aef
			[6]
(e)	Time required $=2000\ln 2-1000 = 386.2943611...$ sec		
	$=386$ seconds (nearest second)		
	$=6$ minutes and 26 seconds (nearest second)	<u>6 minutes, 26 seconds</u>	B1
			[1]
			13 marks

Q5.

Question	Scheme	Marks	AOs
(a)	$\frac{dV}{dt} = 0.48 - 0.1h$	B1	3.1b
	$V = 24h \Rightarrow \frac{dV}{dh} = 24$ or $\frac{dh}{dV} = \frac{1}{24}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.48 - 0.1h}{24}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.48 - 0.1h = 24\frac{dh}{dt}$	M1	2.1
	$1200\frac{dh}{dt} = 24 - 5h$ *	A1*	1.1b
		(4)	
(b)	$1200\frac{dh}{dt} = 24 - 5h \Rightarrow \int \frac{1200}{24-5h}dh = \int dt$ $\Rightarrow$ *e.g.* $\alpha \ln(24-5h) = t(+c)$ oe or $1200\frac{dh}{dt} = 24 - 5h \Rightarrow \frac{dt}{dh} = \frac{1200}{24-5h}$ $\Rightarrow$ *e.g.* $t(+c) = \alpha \ln(24-5h)$ oe	M1	3.1a
	$t = -240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t = 0, h = 2 \Rightarrow 0 = -240\ln(24-10) + c \Rightarrow c = ...(240\ln 14)$	M1	3.4
	$t = 240\ln(14) - 240\ln(24-5h)$	A1	1.1b
	$t = 240\ln\frac{14}{24-5h} \Rightarrow \frac{t}{240} = \ln\frac{14}{24-5h} \Rightarrow e^{\frac{t}{240}} = \frac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}} = 24 - 5h \Rightarrow h = ...$	ddM1	2.1
	$h = 4.8 - 2.8e^{-\frac{t}{240}}$ oe e.g. $h = \frac{24}{5} - \frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, e^{-\frac{t}{240}} \to 0$ • When $h > 4.8$, $\frac{dV}{dt} < 0$ • Flow in = flow out at max h so $0.1h = 4.8 \rightarrow h = 4.8$ • As $e^{-\frac{t}{240}} > 0$, $h < 4.8$ • $h = 5 \Rightarrow \frac{dV}{dt} = -0.02$ or $\frac{dh}{dt} = -\frac{1}{1200}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.8$ • $h = 5 \Rightarrow 4.8 - 2.8e^{-\frac{t}{240}} = 5 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If $h = 5$ the tank would be emptying so can never be full • The equation can't be solved when $h = 5$	A1	3.2a
		(2)	
		(12 marks)	

Q6.

Question Number	Scheme		Marks	
	(a) $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1	
	$V = 9\pi h \Rightarrow \frac{dV}{dt} = 9\pi \frac{dh}{dt}$		B1	
	$9\pi \frac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1	
	Leading to $75\frac{dh}{dt} = 4 - 5h$ *	cso	A1	(5)
	(b) $\int \frac{75}{4-5h}\,dh = \int 1\,dt$	separating variables	M1	
	$-15\ln(4-5h) = t \ (+C)$		M1 A1	
	$-15\ln(4-5h) = t + C$			
	When $t = 0$, $h = 0.2$			
	$-15\ln 3 = C$		M1	
	$t = 15\ln 3 - 15\ln(4-5h)$			
	When $h = 0.5$			
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1	
	Alternative for last 3 marks			
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$			
	$= -15\ln 1.5 + 15\ln 3$		M1 M1	
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	A1	(6)

Q7.

Question Number	Scheme		Marks
(a)	$\frac{dP}{dt} = kP$ and $t=0,\ P=P_0$ (1)		
	$\int \frac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\ dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt; (+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt+\ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 **[4]**
(b)	$P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ ...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399 \text{min}}$ or $t = \underline{6 \text{ hr } 39 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6 \text{ hr } 39 \text{ mins}}$	A1 **[3]**
	$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).		
(c)	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t=0,\ P=P_0$ (1)		
	$\int \frac{dP}{P} = \int \lambda \cos \lambda t\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda \cos \lambda t\, dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin \lambda t; (+c)$	Must see $\ln P$ and $\sin \lambda t$; Correct equation with/without + c.	A1
	When $t=0,\ P=P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1 **[4]**

(d)	$P = 2P_0$ & $\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$			
	$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$ ...or ... $e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda t = \ln 2}$	Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1	
	$\underline{t = \frac{1}{2.5}\sin^{-1}(\ln 2)}$	Then rearranges to make t the subject. (must use $\sin^{-1}$)	dM1	
	$t = 0.306338477...$			
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes			
	$t = \underline{441}$min or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7 \text{ hr } 21 \text{ mins}}$	A1	
			[3]	
			14 marks	

$\underline{P = P_0 e^{\sin \lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Q8.

Question Number	Scheme	Marks
	$\frac{dx}{dt} = k(M - x)$, where M is a constant	
(a)	$\frac{dx}{dt}$ is the rate of increase of the mass of waste products. — Any one correct explanation.	B1
	M is the total mass of unburned fuel and waste fuel (or the initial mass of unburned fuel) — Both explanations are correct.	B1
		[2]
(b)	$\int \frac{1}{M - x}\,dx = \int k\,dt$ or $\int \frac{1}{k(M - x)}\,dx = \int dt$	B1
	$-\ln(M - x) = kt\ \{+c\}$ or $-\frac{1}{k}\ln(M - x) = t\ \{+c\}$ — See notes	M1 A1
	$\{t = 0,\ x = 0 \Rightarrow\}\ -\ln(M - 0) = k(0) + c$ — See notes	M1
	$c = -\ln M \Rightarrow -\ln(M - x) = kt - \ln M$	
	then either... $-kt = \ln(M - x) - \ln M$ / *or...* $kt = \ln M - \ln(M - x)$	
	$-kt = \ln\left(\frac{M - x}{M}\right)$ / $kt = \ln\left(\frac{M}{M - x}\right)$	
	$e^{-kt} = \frac{M - x}{M}$ / $e^{kt} = \frac{M}{M - x}$	ddM1
	$Me^{-kt} = M - x$ / $(M - x)e^{kt} = M$, $M - x = Me^{-kt}$	
	leading to $x = M - Me^{-kt}$ or $x = M(1 - e^{-kt})$ oe	A1 * cso
		[6]
(c)	$\left\{x = \frac{1}{2}M,\ t = \ln 4 \Rightarrow\right\}\ \frac{1}{2}M = M(1 - e^{-k\ln 4})$	M1
	$\Rightarrow \frac{1}{2} = 1 - e^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \frac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$	
	So $k = \frac{1}{2}$	A1
	$x = M\left(1 - e^{-\frac{1}{2}\ln 9}\right)$	dM1
	$x = \frac{2}{3}M$ — $x = \frac{2}{3}M$	A1 cso
		[4]
		12

Q9.

Question Number	Scheme			Marks
(a)	$\frac{dN}{dt} = \frac{(kt-1)(5000-N)}{t}, \quad t>0, \ 0<N<5000$			
	$\int \frac{1}{5000-N}\,dN = \int \frac{(kt-1)}{t}\,dt \quad \left\{\text{or} = \int\left(k-\frac{1}{t}\right)dt\right\}$ See notes			B1
	$-\ln(5000-N) = kt - \ln t; +c$ See notes			M1 A1; A1
	then eg either...	*or...*	*or...*	
	$-kt+c = \ln(5000-N) - \ln t$	$kt+c = \ln t - \ln(5000-N)$	$\ln(5000-N) = -kt + \ln t + c$	
	$-kt+c = \ln\left(\frac{5000-N}{t}\right)$	$kt+c = \ln\left(\frac{t}{5000-N}\right)$	$5000-N = e^{-kt+\ln t+c}$	
	$e^{-kt+c} = \frac{5000-N}{t}$	$e^{kt+c} = \frac{t}{5000-N}$	$5000-N = t\,e^{-kt+c}$	
	leading to $N = 5000 - Ate^{-kt}$ **with no incorrect working/statements. See notes**			A1 * cso [5]
(b)	$\{t=1, N=1200 \Rightarrow\}$ $1200 = 5000 - Ae^{-k}$ $\{t=2, N=1800 \Rightarrow\}$ $1800 = 5000 - 2Ae^{-2k}$ So $Ae^{-k} = 3800$ and $2Ae^{-2k} = 3200$ or $Ae^{-2k} = 1600$	At least one correct statement written down using the boundary conditions		B1
	Eg. $\frac{e^{-k}}{2e^{-2k}} = \frac{3800}{3200}$ or $\frac{2e^{-2k}}{e^{-k}} = \frac{3200}{3800}$ So $\frac{1}{2}e^{k} = \frac{3800}{3200}$ or $2e^{-k} = \frac{3200}{3800}$	An attempt to eliminate A by producing an equation in only k.		M1
	$k = \ln\left(\frac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\frac{19}{8}\right)\right\}$	At least one of $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent		A1
	$\left\{A = 3800(e^{k}) = 3800\left(\frac{19}{8}\right) \Rightarrow\right\}$ $A = 9025$	Both $A = 9025$ cao or $k = \ln\left(\frac{7600}{3200}\right)$ or exact equivalent		A1 [4]
	Alternative Method for the M1 mark in (b)			
	$e^{-k} = \frac{3800}{A}$ $2A\left(\frac{3800}{A}\right)^2 = 3200$	An attempt to eliminate k by producing an equation in only A		M1
(c)	$\left\{t=5, \ N = 5000 - 9025(5)e^{-5\ln\left(\frac{19}{8}\right)}\right\}$ $N = 4402.828401... = 4400$ (fish) (nearest 100)	anything that rounds to 4400		B1 [1] 10

Q10.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathrm{d}h = -2(4-u)\,\mathrm{d}u$	B1	This mark is given for finding an expression for $\mathrm{d}h$
	$\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \int \frac{-2(4-u)\,\mathrm{d}u}{4-\sqrt{h}}$	M1	This mark is given for substituting $u = 4-\sqrt{h}$ into the integral
	$= \int -\frac{8}{u} + 2\,\mathrm{d}u$	M1	This mark is given for a method to find a simplified version of the integral
	$-8\ln u + 2u + c$ $= -8\ln(4-\sqrt{h}) + 2(4-\sqrt{h}) + c$	M1	This mark is given for integrating with respect to u to produce an expression in terms of h
		A1	This mark is given for a correct expression for the integral
	$= -8\ln(4-\sqrt{h}) - 2\sqrt{h} + k$	A1	This mark is given for a full proof to arrive at the answer as shown (appreciating that $k = c + 8$)
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$	M1	This mark is given for a setting $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$
	$0 < h < 16$	A1	This mark is given for deducing the range of the heights of the trees for this model
(c)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20} \Rightarrow \frac{\mathrm{d}h}{(4-\sqrt{h})} = \frac{t^{0.25}\mathrm{d}t}{20}$	B1	This mark is given for separating the variables
	$-8\ln(4-\sqrt{h}) - 2\sqrt{h} + k = \frac{t^{1.25}}{25}$	M1	This mark is given for a method to integrate both sides of the equation
		A1	This mark is given for integrating both sides of the equation correctly
	When $t = 0$ and $h = 1$, $-8\ln 3 - 2 + k = 0$ $k = 2 + 8\ln 3$	M1	This mark is given for substituting values of $t = 0$ and $h = 1$ to find a value for k
	When $h = 12$, $-8\ln(4-\sqrt{12}) - 2\sqrt{12} + 2 + 8\ln 3 = \frac{t^{1.25}}{25}$	M1	This mark is given for a method to find a value for t by substituting $h = 12$ into the equation
	$t^{1.25} = 221.2795 \Rightarrow t = \sqrt[1.25]{221.2795}$	M1	This mark is given for simplifying to find an expression for t
	$t = 75.2$ years	A1	This mark is given for correctly finding the time the tree would take to reach a height of 12 metres
			(Total 15 marks)