Connected Rates Of Change (From OCR 4723)

Q1, (Jan 2007, Q4)

(i) Obtain derivative of form $k(4t+9)^{-\frac{1}{2}}$ — M1 — any constant k

Obtain correct $2(4t+9)^{-\frac{1}{2}}$ — A1 — or (unsimplified) equiv

Obtain derivative of form $k\,e^{\frac{1}{2}x+1}$ — M1 — any constant k different from 6

Obtain correct $3e^{\frac{1}{2}x+1}$ — A1 **4** — or equiv

(ii) Either: Form product of two derivatives M1 — numerical or algebraic

Substitute for t and x in product M1 — using $t = 4$ and calculated value of x

Obtain 39.7 — A1 **3** — allow ±0.1; allow greater accuracy

Or: Obtain $k(4t+9)^n e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$ — M1 — differentiating $y = 6e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$

Obtain correct $6(4t+9)^{-\frac{1}{2}} e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$ — A1 — or equiv

Substitute $t = 4$ to obtain 39.7 — A1 **(3)** — allow ±0.1; allow greater accuracy

Q2, (Jan 2008, Q4)

(i) Obtain derivative of form $kh^5(h^6+16)^n$ — **M1** — any constant k; any $n < \frac{1}{2}$; allow if -4 term retained

Obtain correct $3h^5(h^6+16)^{-\frac{1}{2}}$ — **A1** — or (unsimplified) equiv; no -4 now

Substitute to obtain 10.7 — **A1 3** — or greater accuracy or exact equiv

(ii) Attempt multn or divn using 8 and answer from (i) **M1**

Attempt 8 divided by answer from (i) **M1**

Obtain 0.75 — **A1√ 3** — or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

Q3, (Jan 2010, Q7a)

Either: State or imply either $\frac{dA}{dr} = 2\pi r$ or $\frac{dA}{dt} = 250$ — B1 — or both

Attempt manipulation of derivatives to find $\frac{dr}{dt}$ — M1 — using multiplication / division

Obtain correct $\frac{250}{2\pi r}$ — A1 — or equiv

Obtain 1.6 — A1 **4** — or equiv; allow greater accuracy

Or: Attempt to express r in terms of t — M1 — using $A = 250t$

Obtain $r = \sqrt{\frac{250t}{\pi}}$ — A1 — or equiv

Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$ — M1 — any constant k

Substitute $t = 7.6$ to obtain 1.6 — A1 **(4)** — allow greater accuracy

Q4, (Jan 2011, Q3)

Either:	State or imply $8\pi r$ as derivative	B1		or equiv
	Attempt to connect 12 and their derivative	M1		numerical or algebraic; using multiplication or division
	Obtain $8\pi \times 150 \times 12$ and hence 45000 or 14400π or 14000π	A1	**3**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
Or:	Use $r = 12t$ to show $S = 576\pi t^2$	B1		
	Attempt $\frac{dS}{dt}$ and substitute for t	M1		
	Obtain $1152\pi \times \frac{150}{12}$ and hence 45000 or 14400π or 14000π	A1	**(3)**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
			3	

Q5, (Jun 2012, Q6)

(i)		Attempt use of chain rule	*M1	to obtain derivative of form $kh(3h^2+4)^n$, any non-zero constants k and n condone retention of -8	
		Obtain $9h(3h^2+4)^{\frac{1}{2}}$	A1	or (unsimplified) equiv; no -8 here	
		Substitute 0.6 in attempt at first derivative	M1	dep *M; condone retention of -8 here; implied by their value following wrong derivative if no working seen	
		Obtain 12.17	A1 **[4]**	or greater accuracy	
(ii)		State or imply that $\frac{dh}{dt} = -0.015$ or 0.015	B1	implied by use in calculation with part (i) answer	
		Carry out multiplication of $(\pm)0.015$ and answer from part (i)	M1		
		Obtain 0.18 or -0.18 (whatever this value is claimed to be)	A1 **[3]**	or greater accuracy; condone absence or misuse of negative signs throughout; ignore units; allow for answer rounding to 0.18 following slight inaccuracy due to use of 12.18 or 12.2 or …	

Q6, (Jun 2013, Q3)

(i)		Use α (possibly implicitly) to state that radius of 'base' is $\frac{1}{2}x$	*B1	or to obtain equiv such as $2r = x$ or $\frac{r}{x} = \frac{1}{2}$ or $\frac{x}{r} = 2$
		Substitute into formula to obtain $\frac{1}{3}\pi(\frac{1}{2}x)^2 x$ or $\frac{1}{3}\pi\frac{1}{4}x^2 x$ and obtain $\frac{1}{12}\pi x^3$	B1 [2]	dep *B; AG; necessary detail needed Note: comparing formulae $\frac{1}{3}\pi r^2 h$ and $\frac{1}{12}\pi x^3$ to 'deduce' is B0B0
(ii)		Differentiate to obtain $\frac{1}{4}\pi x^2$ or equiv	B1	whatever they call it
		Attempt division involving 14 and their value of derivative when $x = 8$	M1	ie $14 \div$ deriv or deriv $\div 14$ with $x = 8$
		Obtain 0.28	A1	allow 0.279 but not greater accuracy Alternatives: 1. $14t = \frac{1}{12}\pi x^3$ Obtain $\frac{dt}{dx} = \frac{1}{56}\pi x^2$ B1 Sub 8 and invert M1 Ans A1 2. $x^3 = \frac{168t}{\pi}$ Obtain $3x^2\frac{dx}{dt} = \frac{168}{\pi}$ B1 Sub 8 M1 Ans A1 [3]

Q7, (Jun 2015, Q3)

Differentiate to obtain $kh^n(2+\sqrt{h})^5$	M1	Any non-zero constants k, n; condone presence of -192 here	
Obtain $9h^{-\frac{1}{2}}(2+\sqrt{h})^5$ or unsimplified equiv	A1	Without -192 now	
Divide 150 by their derivative, algebraic or numerical	*M1	Using any recognisable attempt at first derivative	
Substitute $h = 1.4$ and evaluate	M1	Dep *M; assume appropriate substitution if calculation goes wrong	
Obtain 0.06 or 0.060 or 0.0603	A1	But not greater accuracy in final answer; units not needed unless change made to metres and/or hours	
	[5]		

Q8, (Jun 2018, Q6a)

Differentiate V to obtain form $k(5+2x)^2$	M1	M0 if -250 retained in derivative
Obtain correct $12(5+2x)^2$ or (unsimplified) equiv	A1	If V expanded, $300+240x+48x^2$ earns B2, and award B1 if this has only one error
Divide 15 by their attempt at $\frac{dV}{dx}$ with $x=1.6$	M1	Not dependent on first M but attempt at differentiation must have occurred
Obtain 0.019 or 0.0186	A1	Accept greater accuracy; no need for units unless adjustment made in which case units must be clearly shown (e.g. 0.00031 ms^{-1})
	[4]	