



Connected Rates of Change (Sheet 2)

Q1.

The area A of a circle is increasing at a constant rate of $1.5 \text{ cm}^2 \text{ s}^{-1}$. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2 .

(5)

(Total 5 marks)

Q2.

At time t seconds the radius of a sphere is $r \text{ cm}$, its volume is $V \text{ cm}^3$ and its surface area is $S \text{ cm}^2$.

[You are given that $V = \frac{4}{3} \pi r^3$ and that $S = 4\pi r^2$]

The volume of the sphere is increasing uniformly at a constant rate of $3 \text{ cm}^3 \text{ s}^{-1}$.

(a) Find $\frac{dr}{dt}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm.

(2)

(Total 6 marks)

Q3.

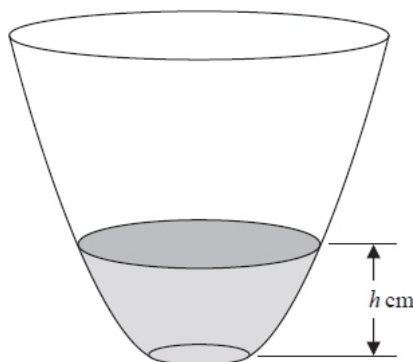


Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is $h \text{ cm}$, the volume of water $V \text{ cm}^3$ is given by

$$V = 4\pi h(h + 4), \quad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of $80\pi \text{ cm}^3 \text{ s}^{-1}$

Find the rate of change of the depth of the water, in cm s^{-1} , when $h = 6$

(5)

(Total 5 marks)



Q4.

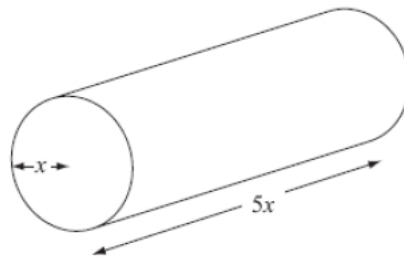


Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.

The cross-sectional area of the rod is increasing at the constant rate of $0.032 \text{ cm}^2 \text{ s}^{-1}$.

(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

(Total 8 marks)

Q5.

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume $V \text{ m}^3$ is given by

$$V = \frac{1}{12} \pi h^2 (3 - 4h), \quad 0 \leq h \leq 0.25$$

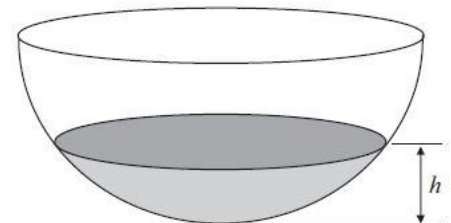


Figure 1

(a) Find, in terms of π , $\frac{dV}{dh}$ when $h = 0.1$

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800} \text{ m}^3 \text{ s}^{-1}$.

(b) Find the rate of change of h , in m s^{-1} , when $h = 0.1$

(2)

(Total 6 marks)



Q6.

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in the diagram. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm³.

(a) Show that $V = \frac{4\pi h^3}{27}$.

(2)

[The volume V of a right circular cone with vertical height h and base radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$.

(b) Find, in terms of π , the rate of change of h when $h = 12$.

(5)

(Total 7 marks)

