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Connected Rates of Change (Sheet 2) Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\frac{dA}{dt} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}}$ $(= 0.797\,884\ldots)$	M1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	
	$1.5 = 2\pi r \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

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Q2.

Question Number	Scheme	Marks
(a)	From question, $V=\frac{4}{3}\pi r^3$, $S=4\pi r^2$, $\frac{dV}{dt}=3$	
	$\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\} \frac{dV}{dr}=4\pi r^2$ $\quad \frac{dV}{dr}=4\pi r^2$ (Can be implied)	B1 oe
	$\left\{\frac{dV}{dr}\times\frac{dr}{dt}=\frac{dV}{dt}\Rightarrow\right\} \quad (4\pi r^2)\frac{dr}{dt}=3$ $\quad \left(\text{Candidate's } \frac{dV}{dr}\right)\times\frac{dr}{dt}=3$	M1 oe
	$\left\{\frac{dr}{dt}=\frac{dV}{dt}\div\frac{dV}{dr}\Rightarrow\right\} \quad \frac{dr}{dt}=(3)\frac{1}{4\pi r^2}\left\{=\frac{3}{4\pi r^2}\right\}$ $\quad$ or $3\div$ Candidate's $\frac{dV}{dr}$;	
	When $r=4\text{cm}$, $\frac{dr}{dt}=\frac{3}{4\pi(4)^2}\left\{=\frac{3}{64\pi}\right\}$ $\quad$ **dependent on previous M1. see notes**	dM1
	Hence, $\frac{dr}{dt}=0.01492077591...(\text{cm}^2\,\text{s}^{-1})$ $\quad$ anything that rounds to 0.0149	A1
		[4]
(b)	$\left\{\frac{dS}{dt}=\frac{dS}{dr}\times\frac{dr}{dt}=\right\}\Rightarrow \frac{dS}{dt}=8\pi r\times\frac{3}{4\pi r^2}\left\{\text{or }\frac{6}{r}\text{ or }8\pi r\times 0.0149...\right\}$ $\quad 8\pi r\times$ Candidate's $\frac{dr}{dt}$	M1; oe
	When $r=4\text{cm}$, $\frac{dr}{dt}=8\pi(4)\times\frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4)\times 0.0149...$	
	Hence, $\frac{dS}{dt}=1.5\ (\text{cm}^2\,\text{s}^{-1})$ $\quad$ anything that rounds to 1.5	A1 cso
		[2]
		6

		Question Notes
(a)		
	B1	$\frac{dV}{dr}=4\pi r^2$ Can be implied by later working.
	M1	$\left(\text{Candidate's } \frac{dV}{dr}\right)\times\frac{dr}{dt}=3$ or $3\div$ Candidate's $\frac{dV}{dr}$
	dM1	**(dependent on the previous method mark)**
		Substitutes $r=4$ into an expression which is a result of a quotient of "3" and their $\frac{dV}{dr}$.
	A1	anything that rounds to 0.0149 (units are not required)
(b)	**M1**	$8\pi r\times$ Candidate's $\frac{dr}{dt}$
	A1	anything that rounds to 1.5 (units are not required). **Correct solution only.**
	Note	Using $\frac{dr}{dt}=0.0149$ gives $\frac{dS}{dt}=1.4979...$ which is fine for A1.

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Q3.

Question Number	Scheme	Marks
	$\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,	
	$\frac{dV}{dh} = 8\pi h + 16\pi$ $\quad \pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $8\pi h + 16\pi$	M1 A1
	$\left\{\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\quad \left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$ $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ $\quad$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$	M1 oe
	When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{= \frac{80\pi}{64\pi}\right\}$ $\quad$ **dependent on the previous M1 see notes** $\frac{dh}{dt} = \underline{1.25}\ (\text{cms}^{-1})$ $\quad$ 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	dM1 A1 oe [5] 5
	Alternative Method for the first M1A1 Product rule: $\begin{cases} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{cases}$ $\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$ $\quad \pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $4\pi(h+4) + 4\pi h$	M1 A1

Question Notes

M1	An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$. **Can be simplified or un-simplified.**
A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
Note	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
M1	$\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$
Note	Also allow 2nd M1 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$
Note	Give 2nd M0 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{dV}{dh}$
dM1	**which is dependent on the previous M1 mark.** Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and 80π (or 80)
A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
Note	Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

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Q4.

Question Number	Scheme	Marks
(a)	From question, $\frac{dA}{dt} = 0.032$	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} =\right\} 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	M1
	When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$	
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	A1 cso (4)
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	B1 ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 . \left(\frac{0.016}{\pi x}\right); \{= 0.24x\}$	M1
	When $x = 2\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ $(\text{cm}^3\,\text{s}^{-1})$	A1 (4)
		(8 marks)

Q5.

Question Number	Scheme		Marks	
	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$	M1 A1	**(4)**
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031	A1	**(2)**
				[6]

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Q6.

Question Number	Scheme		Marks
(a)	Similar triangles $\Rightarrow \frac{r}{h}=\frac{16}{24} \Rightarrow r=\frac{2h}{3}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h=\frac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			(2)
(b)	From the question, $\frac{dV}{dt}=8$	$\frac{dV}{dt}=8$	B1
	$\frac{dV}{dh}=\frac{12\pi h^2}{27}=\frac{4\pi h^2}{9}$	$\frac{dV}{dh}=\frac{12\pi h^2}{27}$ or $\frac{4\pi h^2}{9}$	B1
	$\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh}=8\times\frac{9}{4\pi h^2}=\frac{18}{\pi h^2}$	Candidate's $\frac{dV}{dt}\div\frac{dV}{dh}$;	M1;
		$8\div\left(\frac{12\pi h^2}{27}\right)$ or $8\times\frac{9}{4\pi h^2}$ or $\frac{18}{\pi h^2}$ oe	A1
	When $h=12$, $\frac{dh}{dt}=\frac{18}{144\pi}=\frac{1}{8\pi}$	$\frac{18}{144\pi}$ or $\frac{1}{8\pi}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\frac{18}{144\pi}$.		(5)
			[7]

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