Compound Angle Formulae – Rcos(x) and Rsin(x) (From OCR 4723)

Q1, (Jun 2006, Q8)

(i) Express $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence give details of a pair of transformations which transforms the curve $y = \cos x$ to the curve $y = 5\cos x + 12\sin x$. [3]

(iii) Solve, for $0° < x < 360°$, the equation $5\cos x + 12\sin x = 2$, giving your answers correct to the nearest $0.1°$. [5]

Q2, (Jan 2011, Q4)

(i) Express $24\sin\theta + 7\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence solve the equation $24\sin\theta + 7\cos\theta = 12$ for $0° < \theta < 360°$. **[4]**

Q3, (Jun 2012, Q8)

(i) Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

(a) solve the equation $3\sin\theta + 4\cos\theta + 1 = 0$, giving all solutions for which $-180° < \theta < 180°$, **[4]**

(b) find the values of the positive constants k and c such that

$$-37 \leqslant k(3\sin\theta + 4\cos\theta) + c \leqslant 43$$

for all values of θ. **[4]**

Q4, (Jun 2013, Q8)

(i) Express $4\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

(a) solve the equation $4\cos\theta - 2\sin\theta = 3$ for $0° < \theta < 360°$, **[4]**

(b) determine the greatest and least values of

$$25 - (4\cos\theta - 2\sin\theta)^2$$

as θ varies, and, in each case, find the smallest positive value of θ for which that value occurs. **[5]**

Q5, (Jun 2014, Q9)

(i) Express $5\cos(\theta-60°)+3\cos\theta$ in the form $R\sin(\theta+\alpha)$, where $R>0$ and $0°<\alpha<90°$. **[4]**

(ii) Hence

(a) give details of the transformations needed to transform the curve $y=5\cos(\theta-60°)+3\cos\theta$ to the curve $y=\sin\theta$, **[3]**

(b) find the smallest positive value of β satisfying the equation

$$5\cos\left(\tfrac{1}{3}\beta-40°\right)+3\cos\left(\tfrac{1}{3}\beta+20°\right)=3. \quad \textbf{[5]}$$