

Compound Angle Formulae – Rcos(x) and Rsin(x) (From OCR 4723)

Q1, (Jun 2006, Q8)

<p>(i) State $R = 13$ State at least one equation of form $R \cos \alpha = k$, $R \sin \alpha = k'$, $\tan \alpha = k''$ Obtain 67.4</p>	<p>B1 or equiv M1 or equiv; allow sin / cos muddles; implied by correct α A1 3 allow 67 or greater accuracy</p>
<p>(ii) Refer to translation and stretch State translation in positive x direction by 67.4 State stretch in y direction by factor 13</p>	<p>M1 in either order; allow here equiv terms such as 'move', 'shift'; with both transformations involving constants A1√ or equiv; following their α; using correct terminology now A1√ 3 or equiv; following their R; using correct terminology now</p>
<p>(iii) Attempt value of $\cos^{-1}(2 \div R)$ Obtain 81.15 Obtain 148.5 as one solution Add their α value to second value correctly attempted Obtain 346.2</p>	<p>M1 A1√ following their R; accept 81 A1 accept 148.5 or 148.6 or value rounding to either of these M1 A1 5 accept 346.2 or 346.3 or value rounding to either of these; and no other solutions</p>

Q2, (Jan 2011, Q4)

<p>(i) Obtain $R = 25$ Attempt to find value of α Obtain 16.3°</p>	<p>B1 allow $\sqrt{625}$ or value rounding to 25 M1 implied by correct answer or its complement; allow sin/cos muddles; allow use of radians for this mark; condone $\sin \alpha = 7$, $\cos \alpha = 24$ in the working A1 3 or greater accuracy 16.260...; must be degrees now; allow 16° here</p>

<p>(ii) Show correct process for finding one answer Obtain (28.69 – 16.26 and hence) 12.4° Show correct process for finding second answer Obtain (151.31 – 16.26 and hence) 135° or 135.1°</p>	<p>M1 even if leading to answer outside 0 to 360 A1 or greater accuracy 12.425... or anything rounding to 12.4 M1 even if further incorrect answers produced A1 4 or greater accuracy 135.054...; and no other between 0 and 360</p>
<p>[SC: No working shown and 2 correct angles stated - B1 only in part (ii)]</p>	

Q3, (Jun 2012, Q8)

(i)		State $R = 5$ Attempt to find value of α Obtain 53.1	B1 M1 A1 [3]	implied by correct value or its complement allow $\tan^{-1} \frac{4}{3}$
(ii)	(a)	Attempt to find at least one value of $\theta + \alpha$ Obtain 1 correct value of θ (-64.7 or 138) Attempt correct process to find the second value Obtain second value of θ (138 or -64.7)	M1 A1 M1 A1 [4]	(should be -168.5 or -11.5 or 191.5 or ...) allow ± 0.1 in answer and greater accuracy involving a positive value of $\sin^{-1}(-\frac{1}{5})$ and subtraction of their α allow ± 0.1 in answer and greater accuracy; and no others between -180 and 180
(ii)	(b)	Use -1 as minimum or 1 as maximum value of $\sin(\theta + \alpha)$ Relate $-5k + c$ to -37 and $5k + c$ to 43 Attempt solution of pair of linear eqns Obtain $k = 8$ and $c = 3$	*M1 A1 M1 A1 [4]	as equations or inequalities dep *M; must be equations now SC: both $k = 8$ and $c = 3$ obtained with no working or from unconvincing working, award B2 (i.e. max 2/4)

Q4, (Jun 2013, Q8)

(i)		Obtain $R = \sqrt{20}$ or $R = 4.47$ Attempt to find value of α Obtain 26.6	B1 M1 A1 [3]	implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos \alpha = 4, \sin \alpha = 2$ here but not for A1 or greater accuracy 26.565...; with no wrong working seen
(ii)	(a)	Show correct process for finding one answer Obtain 21.3 Show correct process for finding second answer Obtain 286 or 285.6	M1 A1FT M1 A1FT [4]	allowing for case where the answer is negative or greater accuracy 21.3045...; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong R ie attempting fourth quadrant value minus α value or greater accuracy 285.5653...; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong R ; and no others between 0° and 360°
(ii)	(b)	State greatest value is 25 Obtain value 63.4 clearly associated with correct greatest value State least value is 5 Attempt to find θ from $\cos(\theta + \text{their } \alpha) = -1$ Obtain 153 or 153.4	B1 B1FT B1 M1 A1FT [5]	allow if α incorrect or greater accuracy 63.4349...; following a wrong value of α allow if α incorrect and clearly associated with correct least value or greater accuracy 153.4349...; following a wrong value of α

Q5, (Jun 2014, Q9)

(i)		<p>Simplify to obtain $\frac{11}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$</p> <p>Attempt correct process to find R</p> <p>Attempt correct process to find α</p> <p>Obtain $7\sin(\theta + 51.8)$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>or equiv with two terms perhaps with $\sin 60$ retained</p> <p>for expression of form $a\cos\theta + b\sin\theta$</p> <p>for expression of form $a\cos\theta + b\sin\theta$; condone $\sin\alpha = \frac{11}{2}$, $\cos\alpha = \frac{5}{2}\sqrt{3}$</p> <p>or greater accuracy 51.786...</p>
(ii)	(a)	<p>State stretch and translation in either order</p> <p>State stretch parallel to y-axis with factor $\frac{1}{7}$</p> <p>State translation parallel to θ-axis or x-axis by 51.8 in positive direction or state translation by vector $\begin{pmatrix} 51.8 \\ 0 \end{pmatrix}$</p>	<p>M1</p> <p>A1ft</p> <p>A1ft</p> <p>[3]</p>	<p>or equiv but using correct terminology, not move, squash, ...</p> <p>following their R and clearly indicating correct direction</p> <p>following their α and clearly indicating correct direction; or equiv such as 308.2 parallel to x-axis in negative direction</p>
	(b)	<p>State left-hand side (their R) $\sin(\frac{1}{3}\beta + \gamma)$ where $\gamma \neq \pm(\text{their } \alpha)$, $\gamma \neq \pm 40$, $\gamma \neq \pm 20$</p> <p>Obtain (their R) $\sin(\frac{1}{3}\beta + \text{their } \alpha + 20) = 3$</p> <p>Attempt correct process to find any value of $\frac{1}{3}\beta$</p> <p>Attempt complete process to find positive value of β</p> <p>Obtain 248 or 249 or 248.5</p>	<p>M1</p> <p>A1ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>or equiv such as stating $\theta = \frac{1}{3}\beta + 20$ (and, in this case, allowing A1ft provided value of $\frac{1}{3}\beta$ attempted later)</p> <p>for equation of form $\sin(\frac{1}{3}\beta + \gamma) = k$ where $k < 1$, $k \neq 0$</p> <p>including choosing second quadrant value of their $\sin^{-1}\frac{3}{7}$</p> <p>or greater accuracy 248.508...</p>