Compound Angle Formulae – Rcos(x) and Rsin(x) Exam Questions Sheet 2

Q1.

$$g(\theta) = 4\cos 2\theta + 2\sin 2\theta$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to 2 decimal places.

(3)

(b) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k.

(2)

(Total for question = 10 marks)

Q2.

(a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + a)$, where R and a are constants, $R > 0$ and $0 < a$ 90° Give the exact value of R and give the value of a to 2 decimal places.

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

(Total for question = 10 marks)

Q3.

(a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(2)

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

(5)

(Total 11 marks)

Q4.

(a) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \pi/2$

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta < \pi$, at which this minimum value occurs.

(3)

(Total 9 marks)

Q5.

$f(x) = 7\cos 2x - 24\sin 2x$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the value of R and the value of α.

(3)

(b) Hence solve the equation

$7\cos 2x - 24\sin 2x = 12.5$

for $0 \leq x < 180°$, giving your answers to 1 decimal place.

(5)

(c) Express $14\cos^2 x - 48\sin x \cos x$ in the form $a \cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found.

(2)

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$14\cos^2 x - 48\sin x \cos x$

(2)

(Total 12 marks)

Q6.

(a) Express 3 cos θ + 4 sin θ in the form R cos($\theta - \alpha$), where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of 3 cos θ + 4 sin θ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, f (t), of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

(Total 12 marks)

Q7.

(a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of 2 sin θ − 1.5 cos θ.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leqslant t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

(Total 15 marks)

Q8.

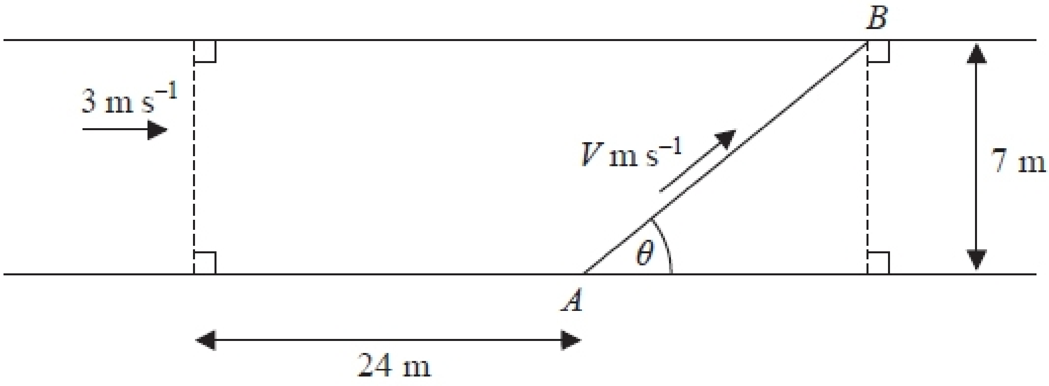

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point *A*.
John passes her as she reaches the other side of the road at a variable point *B*, as shown in Figure 2.
Kate's speed is Vm s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha$ α decimal places.

(3)

Given that θ varies,

(b) find the minimum value of V.

(2)

Given that Kate's speed has the value found in part (b),

(c) find the distance *AB*.

(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$.

(6)

(Total 14 marks)