



**Compound Angle Formulae – Rcos(x) and Rsin(x) Exam Questions Sheet 2**

Q1.

Question Number	Scheme	Marks
(a)	$4 \cos 2\theta + 2 \sin 2\theta = R \cos(2\theta - \alpha)$ $R = \sqrt{4^2 + 2^2} = \sqrt{20} = (2\sqrt{5})$ $\alpha = \arctan\left(\frac{1}{2}\right) = 26.565^\circ \dots = \text{awrt } 26.57^\circ$	B1 M1A1 (3)
(b)	$\sqrt{20} \cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$ $\Rightarrow (2\theta - 26.57) = +77.1 \dots \Rightarrow \theta = \dots$ $\theta = \text{awrt } 51.8^\circ$ $2\theta - 26.57 = '-77.1 \dots' \Rightarrow \theta = -\text{awrt } 25.3^\circ$	M1 dM1 A1 ddM1A1 (5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both (2)
		<b>(10 marks)</b>

Q2.

Question	Scheme	Marks
(a)	$R = \sqrt{5}$ $\tan \alpha = \frac{1}{2} \Rightarrow \alpha = 26.57^\circ$	B1 M1A1 (3)
(b)	$\frac{2}{2 \cos \theta - \sin \theta - 1} = 15 \Rightarrow \frac{2}{\sqrt{5} \cos(\theta + 26.6^\circ) - 1} = 15$ $\Rightarrow \cos(\theta + 26.6^\circ) = \frac{17}{15\sqrt{5}} = (\text{awrt } 0.507)$ $\theta + 26.57^\circ = 59.54^\circ$ $\Rightarrow \theta = \text{awrt } 33.0^\circ \text{ or } \text{awrt } 273.9^\circ$ $\theta + 26.6^\circ = 360^\circ - \text{their } '59.5^\circ'$ $\Rightarrow \theta = \text{awrt } 273.9^\circ \text{ and } \text{awrt } 33.0^\circ$	M1A1 A1 dM1 A1 (5)
(c)	$\theta - \text{their } 26.57^\circ = \text{their } 59.54^\circ \Rightarrow \theta = \dots$ $\theta = \text{awrt } 86.1^\circ$	M1 A1 (2)
		<b>(10 marks)</b>



Q3.

Question Number	Scheme	Marks
(a)	Complete method for $R$ : e.g. $R\cos\alpha = 3$ , $R\sin\alpha = 2$ , $R = \sqrt{3^2 + 2^2}$ $R = \sqrt{13}$ or 3.61 (or more accurate)	M1 A1
	Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$ ] $\alpha = 0.588$ (Allow $33.7^\circ$ )	M1 A1 (4)
(b)	Greatest value = $(\sqrt{13})^4 = 169$	M1, A1 (2)
(c)	$\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ (= 0.27735...) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
	$(x + 0.588) = 0.281(03\dots)$ or $16.1^\circ$	A1
	$(x + 0.588) = \pi - 0.28103\dots$	M1
	Must be $\pi - \text{their } 0.281$ or $180^\circ - \text{their } 16.1^\circ$	
	or $(x + 0.588) = 2\pi + 0.28103\dots$	M1
	Must be $2\pi + \text{their } 0.281$ or $360^\circ + \text{their } 16.1^\circ$	
	$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only)	A1 (5)
	If 0.281 or $16.1^\circ$ not seen, correct answers imply this A mark	(11 marks)

Q4.

Question Number	Scheme	Marks
(a)	$R = \sqrt{20}$	B1
	$\tan\alpha = \frac{4}{2} \Rightarrow \alpha = \text{awrt } 1.107$	M1A1 (3)
(b)(i)	' $4 + 5R^2$ ' = 104	B1ft
(ii)	$3\theta - '1.107' = \frac{\pi}{2} \Rightarrow \theta = \text{awrt } 0.89$	M1A1 (3)
(c)(i)	4	B1
(ii)	$3\theta - '1.107' = 2\pi \Rightarrow \theta = \text{awrt } 2.46$	M1A1 (3)
		(9 marks)



Q5.

<p>(a)</p> $R=25$ $\tan \alpha = \frac{24}{7} \Rightarrow \alpha = (\text{awrt})73.7^\circ$	<p>B1 M1A1</p>
<p>(b)</p> $\cos(2x + \text{their } \alpha) = \frac{12.5}{\text{their } R}$ $2x + \text{their } \alpha = 60^\circ$ $2x + \text{their } \alpha = \text{their } 300^\circ \text{ or their } 420^\circ \Rightarrow x = ..$ $x = \text{awrt } 113.1^\circ, 173.1^\circ$	<p>(3)</p> <p>M1 A1 M1 A1A1</p> <p>(5)</p>
<p>(c)</p> <p>Attempts to use <math>\cos 2x = 2\cos^2 x - 1</math> <b>AND</b> <math>\sin 2x = 2\sin x \cos x</math> in the expression</p> $14\cos^2 x - 48\sin x \cos x = 7(\cos 2x + 1) - 24\sin 2x$ $= 7\cos 2x - 24\sin 2x + 7$	<p>M1</p> <p>A1</p> <p>(2)</p>
<p>(d)</p> $14\cos^2 x - 48\sin x \cos x = R \cos(2x + \alpha) + 7$ <p>Maximum value = 'R' + 'c'</p> $= 32 \text{ cao}$	<p>M1 A1</p> <p>(2)</p> <p><b>(12 marks)</b></p>



Q6.

Question Number	Scheme	Marks
(a)	$R^2 = 3^2 + 4^2$ $R = 5$ $\tan \alpha = \frac{4}{3}$ $\alpha = 53 \dots^\circ$	M1 A1 M1 A1 awrt $53^\circ$
(b)	Maximum value is 5  At the maximum, $\cos(\theta - \alpha) = 1$ or $\theta - \alpha = 0$ $\theta = \alpha = 53 \dots^\circ$	B1 ft M1 A1 ft ft their $R$ ft their $\alpha$
(c)	$f(t) = 10 + 5 \cos(15t - \alpha)^\circ$ Minimum occurs when $\cos(15t - \alpha)^\circ = -1$ The minimum temperature is $(10 - 5)^\circ = 5^\circ$	M1 A1 ft
(d)	$15t - \alpha = 180$ $t = 15.5$	M1 M1 A1 awrt 15.5
		(4) (3) (2) (3) [12]

Q7.

Question Number	Scheme	Marks
(a)	$R = \sqrt{6.25}$ or 2.5 $\tan \alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	B1 M1A1
(b) (i)	Max Value = 2.5	B1 $\sqrt{\quad}$
(ii)	$\sin(\theta - 0.6435) = 1$ or $\theta - \text{their } \alpha = \frac{\pi}{2}; \Rightarrow \theta = \text{awrt } 2.21$	M1; A1 $\sqrt{\quad}$
(c)	$H_{\text{Max}} = 8.5$ (m) $\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$ or $\frac{4\pi t}{25} = \text{their (b) answer}; \Rightarrow t = \text{awrt } 4.41$	B1 $\sqrt{\quad}$ M1; A1
(d)	$\Rightarrow 6 + 2.5 \sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7; \Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$ $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41 Either $t = \text{awrt } 2.1$ or awrt 6.7 So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517\dots \text{ or } 2.730076\dots\}$ Times = {14:06, 18:43}	M1; M1 A1 A1 ddM1 A1
		(3) (3) (3) (6) [15]

**Subscribe To The Ultimate Study Tool For A-Level Maths At [ALEvelMathsRevision.com/UST](http://ALEvelMathsRevision.com/UST)**



Q8.

Question Number	Scheme	Marks
(a)	$R = \sqrt{(7^2 + 24^2)} = 25$ $\tan \alpha = \frac{24}{7}, \Rightarrow \alpha = \text{awrt } 73.74^\circ$	B1 M1A1 (3)
(b)	maximum value of $24\sin x + 7\cos x = 25$ so $V_{\min} = \frac{21}{25} = (0.84)$	M1A1 (2)
(c)	$\text{Distance } AB = \frac{7}{\sin \theta}, \text{ with } \theta = \alpha$ $\text{So distance} = 7.29\text{m} = \frac{175}{24}\text{m}$	M1, B1 A1 (3)
(d)	$R \cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$ $\theta - \alpha = 60 \Rightarrow \theta = \dots, \theta - \alpha = -60 \Rightarrow \theta = \dots$ $\theta = \text{awrt } 133.7, 13.7$	M1, A1 dM1, dM1 A1, A1 (6)

(14 marks)

**Notes for Question**

- (a)  
B1 25. Accept 25.0 but not  $\sqrt{625}$  or answers that are not exactly 25. Eg 25.0001
- M1 For  $\tan \alpha = \pm \frac{24}{7}$ ,  $\tan \alpha = \pm \frac{7}{24}$ .
- If the value of R is used only accept  $\sin \alpha = \pm \frac{24}{R}$ ,  $\cos \alpha = \pm \frac{7}{R}$
- A1 Accept answers which round to 73.74 – must be in degrees for this mark
- (b)  
M1 Calculates  $V = \frac{21}{\text{their 'R'}}$  NOT - R
- A1 Obtains correct answer.  $V = \frac{21}{25}$  Accept 0.84
- Do not accept if you see incorrect working- ie from  $\cos(\theta - \alpha) = -1$  or the minus just disappearing from a previous line.
- Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate V by the quotient rule (or similar), set  $V' = 0$  to find  $\theta$  and then sub this back into V to find its value.



Notes for Question Continued

(c)

M1 Uses the trig equation  $\sin \theta = \frac{7}{AB}$  with a numerical  $\theta$  to find  $AB = \dots$

B1 Uses  $\theta =$  their value of  $\alpha$  in a trig calculation involving sin. ( $\sin \alpha = \frac{AB}{7}$  is condoned)

A1 Obtains answer  $\frac{175}{24}$  or awrt 7.29

(d)

M1 Substitutes  $V = 1.68$  and their answer to part (a) in  $V = \frac{21}{24 \sin \theta + 7 \cos \theta}$  to get an equation

of the form  $R \cos(\theta \pm \alpha) = \frac{21}{1.68}$  or  $1.68R \cos(\theta \pm \alpha) = 21$  or  $\cos(\theta \pm \alpha) = \frac{21}{1.68R}$ .

Follow through on their  $R$  and  $\alpha$

A1 Obtains  $\cos(\theta \pm \alpha) = 0.5$  oe. Follow through on their  $\alpha$ . It may be implied by later working.

dM1 Obtains one value of  $\theta$  in the range  $0 < \theta < 150$  from inverse cos +their  $\alpha$   
It is dependent upon the first M being scored.

dM1 Obtains second angle of  $\theta$  in the range  $0 < \theta < 150$  from inverse cos +their  $\alpha$   
It is dependent upon the first M being scored.

A1 one correct answer awrt  $\theta = 133.7$  or 13.7 1dp

A1 both correct answers awrt  $\theta = 133.7$  and 13.7 1dp.

Extra solutions in the range loses the last A1.

Answers in radians, lose the first time it occurs. Answers must be to 3dp

For your info  $\alpha = 1.287$ ,  $\theta_1 = 2.334$ ,  $\theta_2 = 0.240$