Compound Angle Formulae Exam Questions (From OCR 4723)

Q1, (Jan 2009, Q9)

(i) By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta.$$ **[4]**

(ii) Hence prove that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$ **[3]**

(iii) Show that the only solutions of the equation

$$1 + \cos 6\theta = 18\cos^2\theta$$

are odd multiples of 90°. **[5]**

Q2, (Jan 2010, Q9)

The value of $\tan 10^\circ$ is denoted by p. Find, in terms of p, the value of

(i) $\tan 55^\circ$, **[3]**

(ii) $\tan 5^\circ$, **[4]**

(iii) $\tan\theta$, where θ satisfies the equation $3\sin(\theta + 10^\circ) = 7\cos(\theta - 10^\circ)$. **[5]**

Q3, (Jun 2011, Q9)

(i) Prove that $\dfrac{\sin(\theta - \alpha) + 3\sin\theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3\cos\theta + \cos(\theta + \alpha)} \equiv \tan\theta$ for all values of α. **[5]**

(ii) Find the exact value of $\dfrac{4\sin 149^\circ + 12\sin 150^\circ + 4\sin 151^\circ}{3\cos 149^\circ + 9\cos 150^\circ + 3\cos 151^\circ}$. **[3]**

(iii) It is given that k is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of k, the equation

$$\frac{\sin(6\theta - 15^\circ) + 3\sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3\cos 6\theta + \cos(6\theta + 15^\circ)} = k.$$ **[4]**

Q4, (Jan 2013, Q9)

(i) Prove that

$$\cos^2(\theta + 45°) - \tfrac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2\theta.$$ **[4]**

(ii) Hence solve the equation

$$6\cos^2(\tfrac{1}{2}\theta + 45°) - 3(\cos\theta - \sin\theta) = 2$$

for $-90° < \theta < 90°$. **[3]**

(iii) It is given that there are two values of θ, where $-90° < \theta < 90°$, satisfying the equation

$$6\cos^2(\tfrac{1}{3}\theta + 45°) - 3(\cos\tfrac{2}{3}\theta - \sin\tfrac{2}{3}\theta) = k,$$

where k is a constant. Find the set of possible values of k. **[3]**

Q5, (Jun 2015, Q9)

It is given that $f(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

(i) Show that $f(\theta) = \cos\theta$. Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8\cos^4\theta - 3.$$ **[6]**

(ii) Hence

(a) determine the greatest and least values of $\dfrac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies, **[3]**

(b) solve the equation

$$\sin(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$. **[4]**

Q6, (Jan 2006, Q9)

(i) By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Determine the greatest possible value of

$$9\sin(\tfrac{10}{3}\alpha) - 12\sin^3(\tfrac{10}{3}\alpha),$$

and find the smallest positive value of α (in degrees) for which that greatest value occurs. [3]

(iii) Solve, for $0° < \beta < 90°$, the equation $3\sin 6\beta \operatorname{cosec} 2\beta = 4$. [6]

Q7, (Jun 2016, Q9)

(i) Show that $\sin 2\theta(\tan\theta + \cot\theta) \equiv 2$. **[4]**

(ii) Hence

(a) find the exact value of $\tan\frac{1}{12}\pi + \tan\frac{1}{8}\pi + \cot\frac{1}{12}\pi + \cot\frac{1}{8}\pi$, **[3]**

(b) solve the equation $\sin 4\theta(\tan\theta + \cot\theta) = 1$ for $0 < \theta < \frac{1}{2}\pi$, **[3]**

(c) express $(1-\cos 2\theta)^2\left(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta\right)^3$ in terms of $\sin\theta$. **[2]**

Q8, (Jun 2017, Q8)

(i) Express

$$3\sin 2\theta\sec\theta + 4\sin 2\theta\,\mathrm{cosec}\,\theta$$

in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[5]**

(ii) Hence solve the equation

$$3\sin(2\beta+20°)\sec(\beta+10°) + 4\sin(2\beta+20°)\,\mathrm{cosec}(\beta+10°) = 3$$

for $0° < \beta < 360°$. **[5]**
