

Compound Angle Formulae Exam Questions (From OCR 4723)

Q1, (Jan 2009, Q9)

- (i) State $\cos 2\theta \cos \theta - \sin 2\theta \sin \theta$ B1
 Use at least one of $\cos 2\theta = 2\cos^2 \theta - 1$
 and $\sin 2\theta = 2\sin \theta \cos \theta$ B1
 Attempt to express in terms of $\cos \theta$ only M1 using correct identities for
 $\cos 2\theta, \sin 2\theta$ and $\sin^2 \theta$
 Obtain $4\cos^3 \theta - 3\cos \theta$ A1 4 AG; necessary detail required

- (ii) Either: State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$ B1
 Use expression for $\cos 3\theta$ and
 attempt expansion M1 for expression of form $\pm 2\cos^2 3\theta \pm 1$
 Obtain $32c^6 - 48c^4 + 18c^2 - 1$ A1 3 AG; necessary detail required
Or: State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$ B1 maybe implied
 Express $\cos 2\theta$ in terms of $\cos \theta$
 and attempt expansion M1 for expression of form $\pm 2\cos^2 \theta \pm 1$
 Obtain $32c^6 - 48c^4 + 18c^2 - 1$ A1 (3) AG; necessary detail required

- (iii) Substitute for $\cos 6\theta$ *M1 with simplification attempted
 Obtain $32c^6 - 48c^4 = 0$ A1 or equiv
 Attempt solution for c of equation M1 dep *M
 Obtain $c^2 = \frac{3}{2}$ and observe no solutions A1 or equiv; correct work only
 Obtain $c = 0$, give at least three specific
 angles and conclude odd multiples of 90 A1 5 AG; or equiv; necessary detail required;
 correct work only

Q2, (Jan 2010, Q9)

- (i) Identify $\tan 55^\circ$ as $\tan(45^\circ + 10^\circ)$ B1 or equiv
 Use correct angle sum formula for $\tan(A+B)$ M1 or equiv
 Obtain $\frac{1+p}{1-p}$ A1 3 with $\tan 45^\circ$ replaced by 1

- (ii) Either: Attempt use of identity for $\tan 2A$ *M1 linking 10° and 5°
 Obtain $p = \frac{2t}{1-t^2}$ A1
 Attempt solution for t of quadratic equation M1 dep *M
 Obtain $\frac{-1 + \sqrt{1+p^2}}{p}$ A1 4 or equiv; and no second expression

- Or (1): Attempt expansion of $\tan(60^\circ - 55^\circ)$ *M1
 Obtain $\frac{\sqrt{3} - \frac{1+p}{1-p}}{1 + \sqrt{3} \frac{1+p}{1-p}}$ A1√ follow their answer from (i)
 Attempt simplification to remove
 denominators M1 dep *M
 Obtain $\frac{\sqrt{3}(1-p) - (1+p)}{1-p + \sqrt{3}(1+p)}$ A1 (4) or equiv

Or (2): State or imply $\tan 15^\circ = 2 - \sqrt{3}$ B1
 Attempt expansion of $\tan(15^\circ - 10^\circ)$ M1 with exact attempt for $\tan 15^\circ$
 Obtain $\frac{2 - \sqrt{3} - p}{1 + p(2 - \sqrt{3})}$ A2 (4)

Or (3): State or imply $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ B1 or exact equiv
 Attempt expansion of $\tan(15^\circ - 10^\circ)$ M1 with exact attempt for $\tan 15^\circ$
 Obtain $\frac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$ A2 (4) or equiv

Or (4): Attempt expansion of $\tan(10^\circ - 5^\circ)$ *M1
 Obtain $t = \frac{p-t}{1+pt}$ A1
 Attempt solution for t of quadratic equation M1 dep *M
 Obtain $\frac{-2 + \sqrt{4 + 4p^2}}{2p}$ A1 (4) or equiv; and no second expression

(iii) Attempt expansion of both sides M1
 Obtain $3\sin\theta\cos 10^\circ + 3\cos\theta\sin 10^\circ =$
 $7\cos\theta\cos 10^\circ + 7\sin\theta\sin 10^\circ$ A1 or equiv
 Attempt division throughout by $\cos\theta\cos 10^\circ$ M1 or by $\cos\theta$ (or $\cos 10^\circ$) only
 Obtain $3t + 3p = 7 + 7pt$ A1 or equiv
 Obtain $\frac{3p-7}{7p-3}$ A1 5 or equiv

Q3, (Jun 2011, Q9)

<p>(i) Use at least one identity correctly Attempt use of relevant identities in single rational expression</p>	<p>B1 angle-sum or angle-difference identity</p> <p>M1 not earned if identities used in expression where step equiv to $\frac{A+B+C}{D+E+F} = \frac{A}{D} + \frac{B}{E} + \frac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos\theta\cos\alpha - \sin\theta\sin\alpha + 3\cos\theta + \cos\theta\cos\alpha + \sin\theta\sin\alpha$)</p>
<p>Obtain $\frac{2\sin\theta\cos\alpha + 3\sin\theta}{2\cos\theta\cos\alpha + 3\cos\theta}$</p>	<p>A1 or equiv but with the other two terms from each of num'r and den'r absent</p>
<p>Attempt factorisation of num'r and den'r</p>	<p>M1</p>
<p>Obtain $\frac{\sin\theta}{\cos\theta}$ and hence $\tan\theta$</p>	<p>A1 5 AG; necessary detail needed</p>

<p>(ii) State or imply form $k \tan 150^\circ$</p>	<p>M1 obtained without any wrong method seen</p>
<p>State or imply $\frac{4}{3} \tan 150^\circ$</p>	<p>A1 or equiv such as $\frac{12 \sin 150^\circ}{9 \cos 150^\circ}$</p>
<p>Obtain $-\frac{4}{9}\sqrt{3}$</p>	<p>A1 3 or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3</p>

<p>(iii) State or imply $\tan 6\theta = k$</p>	<p>B1</p>
<p>State $\frac{1}{6} \tan^{-1} k$</p>	<p>B1</p>
<p>Attempt second value of θ</p>	<p>M1 using $6\theta = \tan^{-1} k + (\text{multiple of } 180)$</p>
<p>Obtain $\frac{1}{6} \tan^{-1} k + 30^\circ$</p>	<p>A1 4 and no other value</p>

12

Q4, (Jan 2013, Q9)

<p>(i)</p>	<p>State $\cos\theta\cos 45 - \sin\theta\sin 45$ Use correct identity for $\sin 2\theta$ or $\cos 2\theta$ Attempt complete simplification of left-hand side Obtain $\sin^2\theta$</p>	<p>B1 or equiv including use of decimal approximation for $\frac{1}{\sqrt{2}}$ B1 must be used; not earned for just a separate statement M1 with relevant identities but allowing sign errors, and showing two terms involving $\sin\theta\cos\theta$ A1 AG; necessary detail needed [4]</p>
<p>(ii)</p>	<p>Use identity to produce equation of form $\sin\frac{1}{2}\theta = c$ Obtain 70.5 or 70.6 Obtain -70.5 or -70.6</p>	<p>M1 condoning single value of constant c here (including values outside the range -1 to 1); M0 for $\sin\theta = c$ unless value(s) are subsequently doubled A1 or greater accuracy 70.528... A1√ or greater accuracy -70.528...; following first answer; and no other answer between -90 and 90; answer(s) only : 0/3 [3]</p>
<p>(iii)</p>	<p>State or imply $6\sin^2\frac{1}{3}\theta = k$ Attempt to relate k to at least $6\sin^2 30^\circ$ Obtain $0 < k < \frac{3}{2}$</p>	<p>B1 M1 A1 condone use of \leq [3]</p>

Q5, (Jun 2015, Q9)

(i)		Use at least one addition formula accurately Obtain $\cos \theta$ State $\cos 4\theta = 2\cos^2 2\theta - 1$ Attempt correct use of relevant formulae to express in terms of $\cos \theta$ Obtain correct unsimplified expression in terms of $\cos \theta$ only Simplify to confirm $8\cos^4 \theta - 3$	M1 A1 B1 M1 A1 A1 [6]	Without substituting values for $\cos 30^\circ$, etc. yet AG; necessary detail needed Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ Or in terms of $\cos \theta$ and $\sin \theta$ e.g. $2(2c^2 - 1)^2 - 1 + 4(2c^2 - 1)$ AG; necessary detail needed
(ii)	(a)	Obtain $\frac{1}{12}$ Substitute 0 for $\cos \theta$ in correct expression Obtain $\frac{1}{4}$	B1 M1 A1 [3]	No need to specify greatest and least
	(b)	State or imply $8\cos^4(3\alpha) - 3 = 1$ Attempt correct method to obtain at least one value of α Obtain 10.9 Obtain 49.1	B1 M1 A1 A1 [4]	Or $2\cos^2 6\alpha + 4\cos 6\alpha - 2 = 0$ Allow for equation of form $\cos^4(3\alpha) = k$ where $0 < k < 1$ or for three-term quadratic equation in $\cos 6\alpha$ Or greater accuracy 10.921... Or greater accuracy 49.078...; and no others between 0 and 60

Q6, (Jan 2006, Q9)

<p>(i) State $\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ Use at least one of $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = 1 - 2 \sin^2 \theta$ Attempt complete process to express in terms of $\sin \theta$ Obtain $3 \sin \theta - 4 \sin^3 \theta$</p>	<p>B1 B1 M1 [using correct identities] A1 4 [AG; all correctly obtained]</p>
<p>(ii) State 3 Obtain expression involving $\sin 10\alpha$ Obtain 9</p>	<p>B1 M1 [allow θ/α confusion] A1 3 [and no other value]</p>
<p>(iii) Recognise $\operatorname{cosec} 2\beta$ as $\frac{1}{\sin 2\beta}$ Attempt to express equation in terms of $\sin 2\beta$ only Attempt to find non-zero value of $\sin 2\beta$ Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$ Attempt correct process to find two values of β Obtain 20.1, 69.9</p>	<p>B1 [allow θ/β confusion] M1 [or equiv involving $\cos 2\beta$] M1 [or of $\cos 2\beta$] A1 [or equiv, exact or approx] M1 [provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$] A1 6 [and no others between 0 and 90]</p>

Q7, (Jun 2016, Q9)

i	Use $\sin 2\theta = 2 \sin \theta \cos \theta$ State $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$ or $\tan \theta + \frac{1}{\tan \theta}$ Simplify using correct identities Obtain 2 correctly	<p>B1 B1 Perhaps as part of expression M1 A1 AG; necessary detail needed [4]</p>
ii a	Obtain expression involving at least one of $\sin \frac{1}{6}\pi$ and $\sin \frac{1}{4}\pi$ Obtain $\frac{2}{\sin \frac{1}{6}\pi} + \frac{2}{\sin \frac{1}{4}\pi}$ Obtain $4 + 2\sqrt{2}$ or exact equiv	<p>M1 A1 Or equiv involving cosecant A1 Answer only is 0/3 [3]</p>
b	Use $\sin 4\theta = 2 \sin 2\theta \cos 2\theta$ Obtain $\cos 2\theta = \frac{1}{4}$ or $\cos^2 \theta = \frac{5}{8}$ or $\sin^2 \theta = \frac{3}{8}$ Obtain 0.659 or 0.66	<p>B1 B1 B1 Or greater accuracy; and no others between 0 and $\frac{1}{2}\pi$; allow 0.21π but not 0.659π; answer only earns 0/3 [3]</p>

c	Express in form $k_1 \sin^4 \theta \times \frac{k_2}{\sin^3 \theta}$	M1	A0 if $(-2 \sin^2 \theta)^2$ involved in simplification
	Obtain $4 \sin^4 \theta \times \frac{8}{\sin^3 \theta}$ and hence $32 \sin \theta$	A1	
	[2]		

Q8, Jun 2017, Q8)

i	Use $\sin 2\theta = 2 \sin \theta \cos \theta$	B1	Must be used not merely stated
	Obtain $6 \sin \theta + 8 \cos \theta$	B1	May be implied
	Obtain $R = 10$	B1	From correct $6 \sin \theta + 8 \cos \theta$
	Attempt appropriate trigonometry to find α	M1	Allow for $\tan \alpha = \frac{6}{8}$ or equiv
	Obtain 53.1°	A1	Or greater accuracy $53.13\dots$; with no errors seen
[5]			
ii	State or imply equation is $10 \sin(\beta + 63.1^\circ) = 3$	B1ft	Following their R and α
	Carry out correct process to find one value of β	M1	Not available for finding negative angle; must involve use of 2nd quadrant angle
	Obtain 99.4° (or 314°)	A1	Or greater accuracy $99.4122\dots^\circ$
	Carry out correct process to find second value of β	M1	Must involve use of '5th' quadrant angle
	Obtain 314° (or 99.4°)	A1	Accept value rounding to 314 providing no error; and no others between 0 and 360
			[Note: Solving $10 \sin(\theta + 53.1^\circ) = 3$ can earn M1 M1 if correct processes followed; if continue to find correct angles by subtracting 10° , A1 A1 available; B1 can be retrospectively given even if answers are wrong]
		[5]	