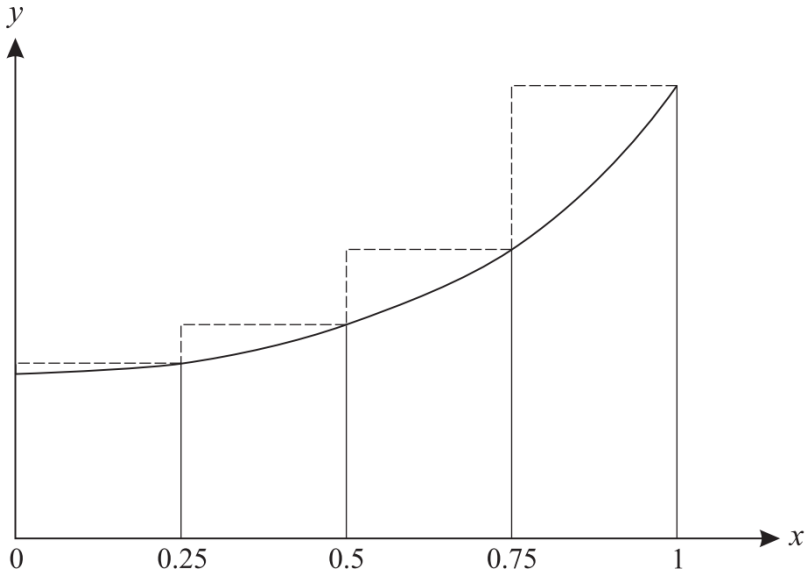
Bounds on Integrals Using Rectangles (From OCR 4726)

Q1, (Jan 2007, Q3)

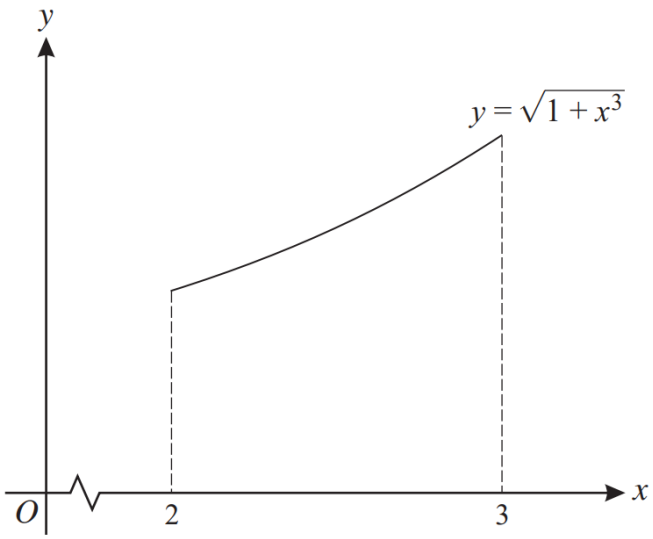


The diagram shows the curve with equation $y = e^{x^2}$, for $0 \leqslant x \leqslant 1$. The region under the curve between these limits is divided into four strips of equal width. The area of this region under the curve is A.

(i) By considering the set of rectangles indicated in the diagram, show that an upper bound for A is 1.71. [3]

(ii) By considering an appropriate set of four rectangles, find a lower bound for A. [3]

Q2, (Jan 2008, Q3)

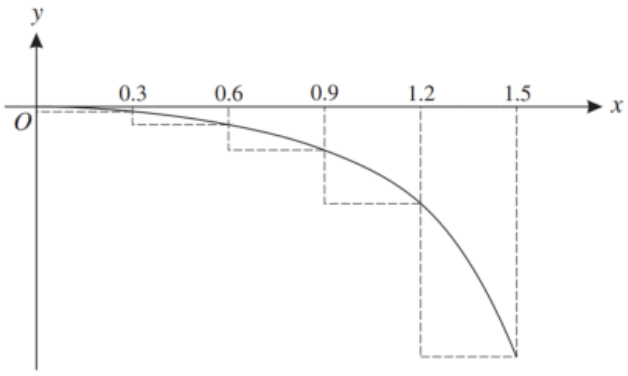


The diagram shows the curve with equation $y = \sqrt{1 + x^3}$, for $2 \leqslant x \leqslant 3$. The region under the curve between these limits has area A.

(i) Explain why $3 < A < \sqrt{28}$. [2]

(ii) The region is divided into 5 strips, each of width 0.2. By using suitable rectangles, find improved lower and upper bounds between which A lies. Give your answers correct to 3 significant figures. [4]

Q3, (Jun 2009, Q1)



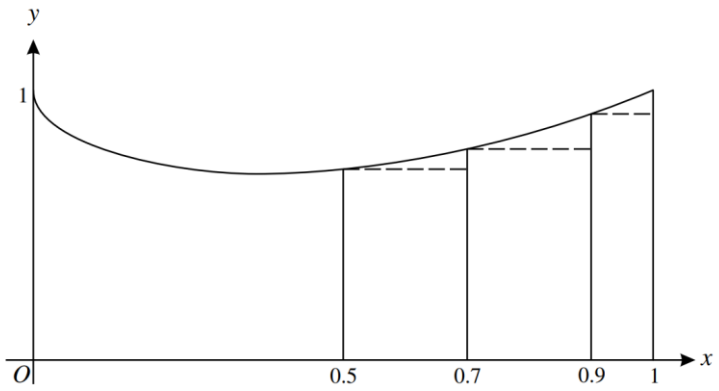
The diagram shows the curve with equation $y = \ln(\cos x)$, for $0 \leqslant x \leqslant 1.5$. The region bounded by the curve, the x-axis and the line $x = 1.5$ has area A. The region is divided into five strips, each of width 0.3.

(i) By considering the set of rectangles indicated in the diagram, find an upper bound for A. Give the answer correct to 3 decimal places. **[2]**

(ii) By considering another set of five suitable rectangles, find a lower bound for A. Give the answer correct to 3 decimal places. **[2]**

(iii) How could you reduce the difference between the upper and lower bounds for A? **[1]**

Q4, (Jan 2011, Q6)



The diagram shows the curve $y = f(x)$, defined by

$$f(x) = \begin{cases} x^x & \text{for } 0 < x \leqslant 1, \\ 1 & \text{for } x = 0. \end{cases}$$

(i) By first taking logarithms, show that the curve has a stationary point at $x = e^{-1}$. **[3]**

The area under the curve from $x = 0.5$ to $x = 1$ is denoted by A.

(ii) By considering the set of three rectangles shown in the diagram, show that a lower bound for A is 0.388. **[2]**

(iii) By considering another set of three rectangles, find an upper bound for A, giving 3 decimal places in your answer. **[2]**

The area under the curve from $x = 0$ to $x = 0.5$ is denoted by B.

(iv) Draw a diagram to show rectangles which could be used to find lower and upper bounds for B, using not more than three rectangles for each bound. (You are not required to find the bounds.) **[3]**
