

Bounds on Integrals Using Rectangles (From OCR 4726)

Q1, (Jan 2007, Q3)

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|---|-------------------------------------|
| (i) Show area of rect. = $\frac{1}{4}(e^{1/16} + e^{1/4} + e^{9/16} + e^1)$ | M1 Or numeric equivalent |
| Show area = 1.7054 | A1 At least 3 d.p. correct |
| Explain the < 1.71 in terms of areas | B1 AG. Inequality required |
| (ii) Identify areas for $>$ sign | B1 Inequality or diagram required |
| Show area of rect. = $\frac{1}{4}(e^0 + e^{1/16} + e^{1/4} + e^{9/16})$ | M1 Or numeric evidence |
| Get $A > 1.27$ | A1 cao; or answer which rounds down |

Q2, (Jan 2008, Q3)

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|---|----|--|
| (i) Get y - values of 3 and $\sqrt{28}$ | B1 | |
| Show/explain areas of two rectangles equal y - value $\times 1$, and relate to A | B1 | Diagram may be used |
| (ii) Show $A > 0.2(\sqrt{(1+2^3)} + \sqrt{(1+2.2^3)} + \dots + \sqrt{(1+2.83)})$ | M1 | Clear areas attempted below curve (5 values) |
| $= 3.87(28)$ | A1 | To min. of 3 s.f. |
| Show $A < 0.2(\sqrt{(1+2.2^3)} + \sqrt{(1+2.4^3)} + \dots + \sqrt{(1+3^3)})$ | M1 | Clear areas attempted above curve (5 values) |
| $= 4.33(11) < 4.34$ | A1 | To min. of 3 s.f. |

Q3, (Jun 2009, Q1)

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|---|----|-------------------------------|
| (i) Attempt area = $\pm \Sigma(0.3y)$ for at least three y values | M1 | May be implied |
| Get 1.313(1..) or 1.314 | A1 | Or greater accuracy |
| (ii) Attempt \pm sum of areas (4 or 5 values) | M1 | May be implied |
| Get 0.518(4..) | A1 | Or greater accuracy |
| | | SC |
| | | If answers only seen, |
| | | 1.313(1..) or 1.314 B2 |
| | | 0.518(4..) B2 |
| | | -1.313(1..) or -1.314 B1 |
| | | -0.518(4..) B1 |
| Or | | |
| Attempt answer to part (i) – final rectangle | M1 | |
| Get 0.518(4..) | A1 | |
| (iii) Decrease width of strips | B1 | Use more strips or equivalent |

Q4, (Jan 2011, Q6)

(i) $y = x^x \Rightarrow \ln y = x \ln x \Rightarrow \frac{1}{y} \frac{dy}{dx} = 1 + \ln x$ M1 For differentiating $\ln y$ OR $x \ln x$ w.r.t. x

$\frac{dy}{dx} = x^x (1 + \ln x) = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$ A1 For $(1 + \ln x)$ seen or implied

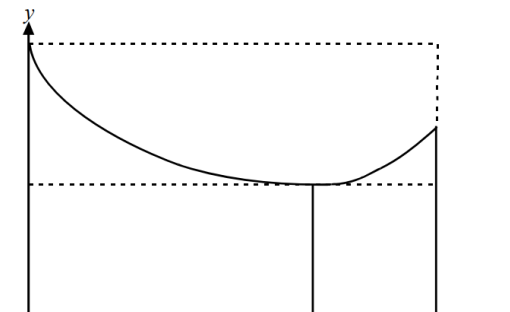
A1 **3** For correct x -value from fully correct working **AG**

(ii) $A > 0.2 \times 0.5^{0.5} + 0.2 \times 0.7^{0.7} + 0.1 \times 0.9^{0.9}$ M1 For areas of 3 lower rectangles

$\Rightarrow A > 0.3881(858) > 0.388$ A1 **2** For lower bound rounding to **AG**

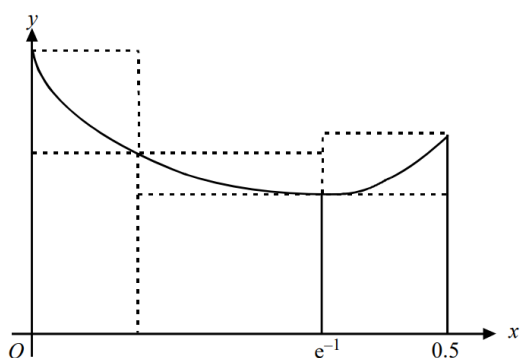
(iii) $A < 0.2 \times 0.7^{0.7} + 0.2 \times 0.9^{0.9} + 0.1 \times 1^1$ M1 For areas of 3 upper rectangles

$\Rightarrow A < 0.4377(177) < 0.438$ A1 **2** For upper bound rounding to 0.438

(iv)  M1 Consider rectangle of height $f(e^{-1})$

A1 Use at least 1 lower rectangle, height $f(e^{-1})$

B1 **3** Use at least 1 upper rectangle, height $f(0)$



SR If more than one rectangle is used for either bound, they must be shown correctly