Question 1 (June 2005, Q1)

Worked Solution

The sequence S has $u_n = 3n - 1$ for $n \geq 1$.

Part (i): Values of u_1, u_2, u_3; type of sequence

$u_1 = 2$, $u_2 = 5$, $u_3 = 8$.

The difference between consecutive terms is constant: $5 - 2 = 3$, $8 - 5 = 3$.

$u_1 = 2$, $u_2 = 5$, $u_3 = 8$. This is an **arithmetic progression** (AP) with common difference 3.

Part (ii): Evaluate $\sum_{n=1}^{100} u_n$

This is an AP with $a = 2$, $d = 3$, $n = 100$.

Using the sum formula $S_n = \frac{n}{2}(2a + (n-1)d)$:

$$S_{100} = \frac{100}{2}(2 \times 2 + 99 \times 3) = 50(4 + 297) = 50 \times 301 = 15050$$

$\sum_{n=1}^{100} u_n = \mathbf{15050}$

Question 2 (Jan 2006, Q1)

Worked Solution

An AP has 20th term = 10 and 50th term = 70.

Part (i): Find the first term and common difference

Using $u_n = a + (n-1)d$:

$$a + 19d = 10 \quad ...(1)$$

$$a + 49d = 70 \quad ...(2)$$

Subtracting (1) from (2): $30d = 60 \Rightarrow d = 2$.

Substituting into (1): $a + 38 = 10 \Rightarrow a = -28$.

$a = -28$, $d = 2$

Part (ii): Show the sum of the first 29 terms is zero

$$S_{29} = \frac{29}{2}(2(-28) + 28 \times 2) = \frac{29}{2}(-56 + 56) = \frac{29}{2} \times 0 = 0$$

$S_{29} = 0$ **(shown)**

Question 3 (Jan 2008, Q6)

Worked Solution

The sequence $u_n = 2n + 5$ for $n \geq 1$.

Part (i): Values of u_1, u_2, u_3

$u_1 = 7$, $u_2 = 9$, $u_3 = 11$.

$u_1 = 7$, $u_2 = 9$, $u_3 = 11$

Part (ii): Type of sequence

Arithmetic progression (common difference $d = 2$)

Part (iii): Find N given $\sum_{n=1}^{N} u_n = 2200$

This is an AP with $a = 7$, $d = 2$:

$$\frac{N}{2}(2\times 7+(N-1)\times 2) = 2200 \implies \frac{N}{2}(14+2N-2) = 2200 \implies \frac{N}{2}(2N+12) = 2200$$

$$N(N+6) = 2200 \implies N^2 + 6N - 2200 = 0$$

Using the quadratic formula (or factorising): $(N-44)(N+50) = 0 \Rightarrow N = 44$ (taking positive value).

$N = \mathbf{44}$

Question 4 (Jan 2013, Q2)

Worked Solution

The sequence: $u_1 = 7$, $u_{n+1} = u_n + 4$ for $n \geq 1$.

This is an AP with $a = 7$, $d = 4$.

Part (i): Show $u_{17} = 71$

$$u_{17} = 7 + 16 \times 4 = 7 + 64 = 71$$

$u_{17} = 71$ **(shown)**

Part (ii): Show $\sum_{n=1}^{35} u_n = \sum_{n=36}^{50} u_n$

$S_{35} = \frac{35}{2}(2 \times 7 + 34 \times 4) = \frac{35}{2}(14 + 136) = \frac{35}{2} \times 150 = 2625$

For $\sum_{n=36}^{50}$, this equals $S_{50} - S_{35}$:

$S_{50} = \frac{50}{2}(2 \times 7 + 49 \times 4) = 25(14 + 196) = 25 \times 210 = 5250$

$S_{50} - S_{35} = 5250 - 2625 = 2625 = S_{35}$ ✓

Both sums equal 2625. **(shown)**

Question 5 (Jun 2009, Q2)

Worked Solution

An AP where the 10th term equals twice the 4th term, and the 20th term is 44.

Part (i): Find the first term and common difference

Using $u_n = a + (n-1)d$:

10th term $= 2\times$ 4th term:

$$a + 9d = 2(a + 3d) \implies a + 9d = 2a + 6d \implies 3d = a \implies a = 3d$$

20th term $= 44$:

$$a + 19d = 44 \implies 3d + 19d = 44 \implies 22d = 44 \implies d = 2, \quad a = 6$$

$a = 6$, $d = 2$

Part (ii): Sum of first 50 terms

$$S_{50} = \frac{50}{2}(2 \times 6 + 49 \times 2) = 25(12 + 98) = 25 \times 110 = 2750$$

$S_{50} = \mathbf{2750}$

Question 6 (Jan 2010, Q8)

Worked Solution

Sequence: $u_1 = 8$, $u_{n+1} = u_n + 3$. This is an AP with $a = 8$, $d = 3$.

Part (i): Show $u_5 = 20$

$u_5 = 8 + 4 \times 3 = 20$ **(shown)**

Part (ii): nth term in the form $u_n = pn + q$

$u_n = 8 + (n-1) \times 3 = 8 + 3n - 3 = 3n + 5$

$p = 3$, $q = 5$ (i.e. $u_n = 3n + 5$)

Part (iii): Type of sequence

Arithmetic progression

Part (iv): Find N such that $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = 1256$

The difference $\sum_{n=1}^{2N} u_n - \sum_{n=1}^{N} u_n = S_{2N} - S_N$.

$$S_{2N} = \frac{2N}{2}(2 \times 8 + (2N-1) \times 3) = N(16 + 6N - 3) = N(6N + 13)$$

$$S_N = \frac{N}{2}(16 + (N-1) \times 3) = \frac{N}{2}(3N + 13)$$

$$S_{2N} - S_N = N(6N+13) - \frac{N}{2}(3N+13) = N\left(6N + 13 - \frac{3N+13}{2}\right) = N \cdot \frac{12N + 26 - 3N - 13}{2} = N \cdot \frac{9N$$

Set equal to 1256:

$$N(9N + 13) = 2512 \implies 9N^2 + 13N - 2512 = 0$$

Using the quadratic formula:

$$N = \frac{-13 + \sqrt{169 + 4 \times 9 \times 2512}}{18} = \frac{-13 + \sqrt{169 + 90432}}{18} = \frac{-13 + \sqrt{90601}}{18} = \frac{-13 + 301}{18} = \frac{288}{18} = 1$$

Check: $16(9 \times 16 + 13) = 16 \times 157 = 2512 = 2 \times 1256$ ✓

$N = \mathbf{16}$

Question 7 (Jan 2011, Q2)

Worked Solution

Sequence S with $u_n = 3n + 2$ for $n \geq 1$.

Part (i): u_1, u_2, u_3

$u_1 = 5$, $u_2 = 8$, $u_3 = 11$.

$u_1 = 5$, $u_2 = 8$, $u_3 = 11$

Part (ii): Type of sequence

Arithmetic progression (common difference $d = 3$)

Part (iii): Find $\sum_{n=101}^{200} u_n$

This equals $S_{200} - S_{100}$.

$a = u_1 = 5$, $d = 3$.

$$S_{200} = \frac{200}{2}(2 \times 5 + 199 \times 3) = 100(10 + 597) = 100 \times 607 = 60700$$

$$S_{100} = \frac{100}{2}(10 + 99 \times 3) = 50(10 + 297) = 50 \times 307 = 15350$$

$$\sum_{n=101}^{200} u_n = 60700 - 15350 = 45350$$

$\sum_{n=101}^{200} u_n = \mathbf{45\,350}$

Question 8 (Jun 2014, Q2)

Worked Solution

Sequence $u_n = 3n - 1$ for $n \geq 1$.

Part (i): Values of u_1, u_2, u_3

$u_1 = 2$, $u_2 = 5$, $u_3 = 8$.

$u_1 = 2,\ u_2 = 5,\ u_3 = 8$

Part (ii): Find $\sum_{n=1}^{40} u_n$

AP with $a = 2$, $d = 3$, $n = 40$:

$$S_{40} = \frac{40}{2}(2 \times 2 + 39 \times 3) = 20(4 + 117) = 20 \times 121 = 2420$$

$\sum_{n=1}^{40} u_n = \mathbf{2420}$

Question 9 (Jun 2015, Q7)

Worked Solution

AP with first term 5, common difference 3. nth term u_n.

Part (i): Find u_{20}

$$u_{20} = 5 + 19 \times 3 = 5 + 57 = 62$$

$u_{20} = \mathbf{62}$

Part (ii): Show $\sum_{n=10}^{20} u_n = 517$

$\sum_{n=10}^{20} u_n = S_{20} - S_9$

$$S_{20} = \frac{20}{2}(10 + 57) = 10 \times 67 = 670$$

$u_{10} = 5 + 9 \times 3 = 32.$

$$S_9 = \frac{9}{2}(10 + 24) = \frac{9}{2} \times 34 = 153$$

$$\sum_{n=10}^{20} u_n = 670 - 153 = 517$$

$\sum_{n=10}^{20} u_n = 517$ **(shown)**

Part (iii): Find N such that $\sum_{n=N}^{2N} u_n = 2750$

$\sum_{n=N}^{2N} u_n = S_{2N} - S_{N-1}$

$$S_{2N} = \frac{2N}{2}(10 + (2N-1) \times 3) = N(6N + 7)$$

$$S_{N-1} = \frac{N-1}{2}(10 + (N-2) \times 3) = \frac{N-1}{2}(3N + 4)$$

$$S_{2N} - S_{N-1} = N(6N+7) - \frac{(N-1)(3N+4)}{2}$$

$$= \frac{2N(6N+7) - (N-1)(3N+4)}{2} = \frac{12N^2 + 14N - 3N^2 - 4N + 3N + 4}{2} = \frac{9N^2 + 13N + 4}{2}$$

Set equal to 2750:

$$9N^2 + 13N + 4 = 5500 \implies 9N^2 + 13N - 5496 = 0$$

Using the quadratic formula:

$$N = \frac{-13 + \sqrt{169 + 4 \times 9 \times 5496}}{18} = \frac{-13 + \sqrt{169 + 197856}}{18} = \frac{-13 + \sqrt{198025}}{18} = \frac{-13 + 445}{18} = \frac{432}{18} =$$

$N = \mathbf{24}$

End of Worked Solutions