Arithmetic Series Exam Questions MS (from OCR 4722)

Q1, (June 2005, Q1)

(i)	$u_1 = 2, u_2 = 5, u_3 = 8$	B1		For the correct value of u_1
		B1		For both correct values of u_2 and u_3
	The sequence is an Arithmetic Progression	B1	3	For a correct statement (any mention of arithmetic)
(ii)	$\frac{1}{2} \times 100 \times (2 \times 2 + 99 \times 3) = 15050$	M1		For correct interpretation of Sigma notation – ie finding the sum of an AP or GP
		M1		For use of correct $\frac{1}{2}n(2a + (n-1)d)$, or equiv, with n=100 and a & d not both =1
		A1	3	For correct value 15050
			6	

Q2, (Jan 2006, Q1)

(i)	$a + 19d = 10, \quad a + 49d = 70$	M1		Attempt to find d from simultaneous equations involving $a + (n-1)d$ or equiv method
	Hence $30d = 60 \Rightarrow d = 2$	A1		Obtain $d = 2$
	$a + (19 \times 2) = 10$ or $a + (49 \times 2) = 70$	M1		Attempt to find a from $a + (n-1)d$ or equiv
	Hence $a = -28$	A1	**4**	Obtain $a = -28$
(ii)	$S = \frac{29}{2}(2 \times -28 + (29-1) \times 2) = 0$	M1		For relevant use of $\frac{1}{2}n(2a + (n-1)d)$
		A1	**2**	For showing the given result correctly **AG**
			6	

Q3, (Jan 2008, Q6)

(i)	$u_1 = 7$	B1		Correct u_1
	$u_2 = 9, u_3 = 11$	B1	**2**	Correct u_2 and u_3
(ii)	Arithmetic Progression	B1	**1**	Any mention of arithmetic
(iii)	½ N (14 + (N – 1) x 2) = 2200	B1		Correct interpretation of sigma notation
		M1		Attempt sum of AP, and equate to 2200
	$N^2 + 6N - 2200 = 0$	A1		Correct (unsimplified) equation
	$(N - 44)(N + 50) = 0$	M1		Attempt to solve 3 term quadratic in N
	hence $N = 44$	A1	**5**	Obtain $N = 44$ only ($N = 44$ www is full marks)
			8	

Q4, (Jan 2013, Q2)

(i)		$7 + 16 \times 4 = 71$ **AG**	M1	Attempt to find 17th term in the given AP
			A1	Show clear detail to obtain $u_{17} = 71$
			[2]	
(ii)		$S_{35} = {}^{35}/_{2}\,(2 \times 7 + 34 \times 4)$ $= 2625$	M1	Attempt sum of first 35 terms of given AP
			A1	Obtain 2625
		either $S_{50} = {}^{50}/_{2}\,(2 \times 7 + 49 \times 4)$ $= 5250$ $5250 - 2625 = 2625$ **AG** or $S_{36-50} = {}^{15}/_{2}\,(2 \times 147 + 14 \times 4)$ $= 2625$ **AG**	M1	Attempt a correct method to show given relationship
			A1	Show given equality convincingly
			[4]	

Q5, (Jun 2009, Q2)

(i)	$a + 9d = 2(a + 3d)$		M1*	Attempt use of $a + (n-1)d$ or $a + nd$ at least once for u_4, u_{10} or u_{20}
	$a = 3d$		A1	Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$
	$a + 19d = 44 \Rightarrow 22d = 44$		M1dep*	Attempt to eliminate one variable from two simultaneous equations in a and d, from u_4, u_{10}, u_{20} and no others
	$d = 2$, $a = 6$		A1 **4**	Obtain $d = 2$, $a = 6$
(ii)	$S_{50} = {}^{50}/_{2}\,(2\text{x}6 + 49\text{x}2)$		M1	Attempt S_{50} of AP, using correct formula, with $n = 50$, allow $25(2a + 24d)$
	$= 2750$		A1 **2**	Obtain 2750
			6	

Q6, (Jan 2010, Q8)

(i) $u_5 = 8 + 4 \times 3$	M1		Attempt $a + (n-1)d$ or equiv inc list of terms
$= 20$ **A.G.**	A1	**2**	Obtain 20
(ii) $u_n = 3n + 5$ ie $p = 3$, $q = 5$	B1		Obtain correct expression, poss unsimplified, eg $8 + 3(n-1)$
	B1	**2**	Obtain correct $3n + 5$, or $p = 3$, $q = 5$ stated
(iii) arithmetic progression	B1	**1**	Any mention of arithmetic
(iv) $\frac{2N}{2}(16 + (2N-1)3) - \frac{N}{2}(16 + (N-1)3) = 1256$	M1		Attempt S_N, using any correct formula (inc $\sum (3n + 5)$)
	M1		Attempt S_{2N}, using any correct formula, with $2N$ consistent (inc $\sum (3n + 5)$)
$26N + 12N^2 - 13N - 3N^2 = 2512$			
$9N^2 + 13N - 2512 = 0$	M1*		Attempt subtraction (correct order) and equate to 1256
$(9N + 157)(N - 16) = 0$	M1dep*		Attempt to solve quadratic in N
$N = 16$	A1	**5**	Obtain $N = 16$ only, from correct working
	OR: alternative method is to use $^n/_2\ (a + l) = 1256$		
	M1		Attempt given difference as single summation with N terms
	M1		Attempt $a = u_{N+1}$
	M1		Attempt $l = u_{2N}$
	M1		Equate to 1256 and attempt to solve quadratic
	A1		Obtain $N = 16$ only, from correct working
		10	

Q7, (Jan 2011, Q2)

(i)	$u_1 = 5, u_2 = 8, u_3 = 11$	**B1**		Obtain at least one correct term	Just a list of numbers is fine, no need for labels.
		B1	**2**	Obtain all three correct terms	Ignore extra terms beyond u_3.
(ii)	arithmetic progression	**B1**	**1**	Any mention of arithmetic	Allow AP, but not description eg constant difference. Ignore extra description eg diverging as long as not wrong or contradictory.
(iii)	$S = {}^{100}/_{2}\,(305 + 602)$ or ${}^{100}/_{2}(2\text{x}305 + 99\text{x}3)$ $= 45{,}350$ (or $S_{200} - S_{100} = 60{,}700 - 15{,}350$)	**M1**		Attempt relevant S_n using correct formula	Must use correct formula to sum an AP – only exception is using $(\frac{1}{2}n - 1)d$ rather than $(n - 1)d$. Must use $d = 3$ (or their d from (i) as long as constant difference). If (i) is incorrect they can still get full marks in (iii) as independent. They need to be finding the sum of 99, 100, 101 or 200 terms and make a reasonable attempt at a value of a consistent with their n – if $n = 99$ then $a = 305$ / if $n = 100$ then $a = 5$ or $a = 305$ / if $n = 101$ then $a = 5$ / if $n = 200$ then $a = 5$. Allow slips on $a = 305$ as long as clearly intending to find u_{101}. If using $\frac{1}{2}n(a + l)$ then there also needs to be a reasonable attempt at l. Attempting to sum from $n = 101$ to $n = 200$ gets both method marks together (assuming that the attempt satisfies above conditions).
		M1		Attempt correct method to find required sum	$S_{200} - S_{101}$ is M0. M0 M1 is possible for correct method but with incorrect formula for S_N (but must be recognisable as attempt at sum of AP). Need to show subtraction to gain M1, just calculating two relevant sums is not yet enough. Still need $a = 5$ and $d = 3$.
		A1	**3**	Obtain 45,350	Answer only gets full marks.
			6		**SR:** if candidates attempt to manually add terms… **M1** Attempt to sum all terms from u_{101} to u_{200} **A2** Obtain 45,350

Q8, (Jun 2014, Q2)

(i)	2, 5, 8	B1		Obtain at least one correct value	Either stated explicitly or as part of a longer list, but must be in correct position eg -1, 2, 5 is B0
		B1		Obtain all three correct values	Ignore any subsequent values, if given
		[2]			
(ii)	$S_{40} = \frac{40}{2}(2 \times 2 + 39 \times 3)$	B1*		Identify AP with $a = 2$, $d = 3$	Could be stated, listing of further terms linked by '+' sign or by recognisable attempt at any formula for AP including attempt at u_{40}
	$= 2420$	M1d*		Attempt to sum first 40 terms of the AP	Must use correct formula, with $a = 2$ and $d = 3$ If using $\frac{1}{2}n(a + l)$ then must be valid attempt at l Could use $3\sum n - \sum 1$, but M0 for $3\sum n - 1$ If summing manually then no need to see all middle terms explicitly as long as intention is clear
		A1		Obtain 2420	Either from formula or from manual summing of 40 terms
		[3]			

Q9(Jun 2015, Q7)

(i)		$u_{20} = 5 + 19 \times 3$	M1	Attempt u_{20}	Must be using correct formula, with $a = 5$ and $d = 3$ Could use $u_n = 3n + 2$ Could attempt to list terms
		$= 62$	A1 **[2]**	Obtain 62	If listing terms then need to indicate that 62 is the required answer
(ii)		$S_{20} = {}^{20}/_{2}\,(10 + 57)$ $S_9 = {}^{9}/_{2}\,(10 + 24)$	M1	Explicitly attempt either S_{20} or S_9	Must be using correct formula with $a = 5$ and $d = 3$ Use of formula must be explicit, so M0 for eg $S_{20} = 670$ with no other evidence Could use ${}^{1}/_{2}n\,(a + l)$, with l obtained from $a + (n - 1)d$ - expect to see ${}^{20}/_{2}\,(5 + 62)$ and/or ${}^{9}/_{2}\,(5 + 29)$ Could use $\Sigma(3n + 2)$, with correct formulae for Σn and $\Sigma 1$
		${}^{20}/_{2}\,(10 + 57) - {}^{9}/_{2}\,(10 + 24)$	M1	Attempt $S_{20} - S_9$, where both summations have been shown explicitly	Can get M1 if formulae have not yet been evaluated M0 for $S_{20} - S_{10}$ (see below for one exception)
		$= 670 - 153$ $= 517$ **AG**	A1	Evaluate both summations and hence obtain 517 CWO	**AG** so detail is required - only award A1 if both unsimplified sums are seen, as well as both evaluated sums **SR** Allow **B1** if only $670 - 153 = 517$ seen Explicitly detailing only one summation will get M1M0A0 Allow 3/3 for $S_{20} - S_{10} + u_{10}$ as long as all explicit Allow 3/3 for manually summing terms as long as all terms are shown and are all correct , but no partial credit if wrong
			[3]		
		OR			
		$u_{10} = 5 + 9 \times 3 = 32$	M1	Attempt u_{10}	Must be shown explicitly
		$S = {}^{11}/_{2}\,(32 + 62)$	M1	Attempt required sum	Must have $n = 11$ Or $S = {}^{11}/_{2}\,(2\times 32 + 10\times 3)$
		$= 517$ **AG**	A1	Obtain 517	Detail reqd - award M0M1A0 if no evidence for $u_{10} = 32$