Question 1

Worked Solution

A company produces 140 bicycles per week, increasing by d each week. In week 12 they produce 206.

Part (a): Find d

Using $u_{12} = a + 11d$ with $a = 140$:

$$206 = 140 + 11d \implies 11d = 66 \implies d = 6$$

$d = \mathbf{6}$

Part (b): Total bicycles in first 52 weeks

Weeks 1 to 12: AP with $a = 140$, $d = 6$, $n = 12$.

$$S_{12} = \frac{12}{2}(140 + 206) = 6 \times 346 = 2076$$

Weeks 13 to 52: 40 weeks at 206 per week.

$$40 \times 206 = 8240$$

Total $= 2076 + 8240 = 10316$.

Total bicycles $= \mathbf{10\,316}$

Question 2

Worked Solution

A shop: 150 computers sold in 2000, increasing by 10 per year.

Part (a): Show 220 computers sold in 2007

In year 2007, $n = 8$ (2007 is 7 years after 2000, so the 8th term with $n = 1$ in 2000):

$$u_8 = 150 + 7 \times 10 = 220 \quad \textbf{(shown)}$$

Part (b): Total computers 2000–2013 (14 years)

AP with $a = 150$, $d = 10$, $n = 14$:

$$S_{14} = \frac{14}{2}(2 \times 150 + 13 \times 10) = 7(300 + 130) = 7 \times 430 = 3010$$

Total computers = **3010**

Part (c): Find the year when selling price (£) $= 3\times$ number sold

In year n (with $n = 1$ in 2000):

- Number sold $= 150 + (n - 1) \times 10$
- Selling price $= 900 - (n - 1) \times 20$

Setting price $= 3\times$ number sold:

$$900 - 20(n - 1) = 3[150 + 10(n - 1)]$$

$$900 - 20(n - 1) = 450 + 30(n - 1)$$

$$450 = 50(n - 1) \implies n - 1 = 9 \implies n = 10$$

$n = 10$ corresponds to the year $2000 + 9 = 2009$.

The year is **2009**

Question 3

Worked Solution

Lewis: AP with $a = 140$, $d = 20$ (spaceships). Sian: AP with $a = 300$, last term $l = 700$, total $S_n = 8500$.

Part (a): Points for Lewis's 20th spaceship

$$T_{20} = 140 + 19 \times 20 = 140 + 380 = 520$$

Lewis scored **520** points for his 20th spaceship.

Part (b): Total for Lewis's first 20 spaceships

$$S_{20} = \frac{20}{2}(140 + 520) = 10 \times 660 = 6600$$

Lewis's total = **6600** points

Part (c): Find n for Sian

Using $S_n = \frac{n}{2}(a + l)$:

$$8500 = \frac{n}{2}(300 + 700) = \frac{n}{2}(1000) = 500n \implies n = 17$$

$n = \mathbf{17}$

Question 4

Worked Solution

Boy's savings: AP with $a = 10$p, $d = 5$p. Sister's savings: AP with $a = 10$p, $d = 10$p.

Part (a): Boy's savings in week 15

$$T_{15} = 10 + 14 \times 5 = 10 + 70 = 80\text{p} = £0.80$$

Boy saves **80**p in week 15.

Part (b): Boy's total over 60 weeks

$$S_{60} = \frac{60}{2}(2 \times 10 + 59 \times 5) = 30(20 + 295) = 30 \times 315 = 9450\text{p} = £94.50$$

Total savings = **£94.50**

Part (c): Show $m(m+1) = 35 \times 36$

Sister's AP: $a = 10$p, $d = 10$p, total $= £63 = 6300$p.

$$S_m = \frac{m}{2}(2 \times 10 + (m-1) \times 10) = \frac{m}{2}(10m + 10) = 5m(m+1)$$

Set $5m(m+1) = 6300$:

$$m(m+1) = 1260 = 35 \times 36 \quad \textbf{(shown)}$$

Part (d): Value of m

$m = \mathbf{35}$ (since $35 \times 36 = 1260$)

Question 5

Worked Solution

John: AP with $a = £60$, $d = £15$ (annual gifts from 10th birthday).

Part (a): Show total after 12th birthday is £225

After 12th birthday, 3 gifts received: $60, 75, 90$.

$$S_3 = 60 + 75 + 90 = 225 \quad \textbf{(shown)}$$

Part (b): Gift on 18th birthday

18th birthday is the 9th gift ($n = 9$):

$$t_9 = 60 + 8 \times 15 = 60 + 120 = £180$$

Gift on 18th birthday = **£180**

Part (c): Total gifts up to and including 21st birthday

21st birthday is the 12th gift ($n = 12$):

$$S_{12} = \frac{12}{2}(2 \times 60 + 11 \times 15) = 6(120 + 165) = 6 \times 285 = £1710$$

Total = **£1710**

Part (d): Show $n^2 + 7n = 25 \times 18$

Set $S_n = 3375$:

$$\frac{n}{2}(2\times60+(n-1)\times15) = 3375 \implies \frac{n}{2}(120+15n-15) = 3375 \implies \frac{n}{2}(15n+105) = 3375$$

$$\frac{15n(n+7)}{2} = 3375 \implies 15n(n+7) = 6750 \implies n(n+7) = 450 = 25 \times 18$$

$$n^2 + 7n = 25 \times 18 \quad \textbf{(shown)}$$

Part (e): Find n; determine John's age

$n^2 + 7n - 450 = 0 \Rightarrow (n - 18)(n + 25) = 0 \Rightarrow n = 18$.

John's age $= 10 + 18 - 1 = 27$ (first gift on 10th birthday, so nth gift on $(10 + n - 1)$th birthday).

$n = 18$; John is **27** years old.

Question 6

Worked Solution

AP: first term 16, 21st term 24.

Part (a): Find the common difference

$$16 + 20d = 24 \implies 20d = 8 \implies d = 0.4$$

$d = \mathbf{0.4}$

Part (b): Sum of first 500 terms

$$S_{500} = \frac{500}{2}(2 \times 16 + 499 \times 0.4) = 250(32 + 199.6) = 250 \times 231.6 = 57900$$

$S_{500} = \mathbf{57\,900}$

Question 7

Worked Solution

Sue's Saturday runs: AP with $a = 5$ km, $d = 2$ km.

Part (a): Show run on 4th Saturday is 11 km

$u_4 = 5 + 3 \times 2 = 11$ km **(shown)**

Part (b): Expression for nth Saturday

$$u_n = 5 + 2(n - 1) = 2n + 3$$

$u_n = 2n + 3$ km

Part (c): Show total in n weeks is $n(n + 4)$ km

$$S_n = \frac{n}{2}(2 \times 5 + (n - 1) \times 2) = \frac{n}{2}(10 + 2n - 2) = \frac{n}{2}(2n + 8) = n(n + 4) \quad \textbf{(shown)}$$

Part (d): Find n when Sue runs 43 km on nth Saturday

$$2n + 3 = 43 \implies 2n = 40 \implies n = 20$$

$n =$ **20**

Part (e): Total distance in $n = 20$ weeks

$$S_{20} = 20(20 + 4) = 20 \times 24 = 480 \text{ km}$$

Total distance = **480** km

Question 8

Worked Solution

Jess: AP with $a = £17\,000$, $d = £1500$, maximum salary $£32\,000$. Total career 20 years.

Part (a): Find k

$$32000 = 17000+(k-1)\times1500 \implies (k-1)\times1500 = 15000 \implies k-1 = 10 \implies k = 11$$

$k = \mathbf{11}$

Part (b): Total earnings over 20 years

Years 1–11: AP with $a = 17000$, $d = 1500$, $n = 11$.

$$S_{11} = \frac{11}{2}(2 \times 17000 + 10 \times 1500) = \frac{11}{2}(34000 + 15000) = \frac{11}{2} \times 49000 = £269500$$

Years 12–20: 9 years at $£32000$.

$$9 \times 32000 = £288000$$

Total $= 269500 + 288000 = £557\,500$.

Total earnings $= \mathbf{£557\,500}$

Question 9

Worked Solution

Xin: first day A minutes, common difference $(d + 1)$. Yi: first day $(A - 13)$ minutes, common difference $(2d - 1)$.

Part (a): Show Xin runs $A + 13d + 13$ minutes on day 14

Day 14 is the 14th term of Xin's AP:

$$u_{14} = A + 13(d + 1) = A + 13d + 13 \quad \textbf{(shown)}$$

Part (b): Find d

Yi's day 14: $(A - 13) + 13(2d - 1) = A - 13 + 26d - 13 = A + 26d - 26$.

Setting Xin's day 14 equal to Yi's day 14:

$$A + 13d + 13 = A + 26d - 26 \implies 13 + 26 = 26d - 13d \implies 39 = 13d \implies d = 3$$

$d = \mathbf{3}$

Part (c): Find A

With $d = 3$, Xin's common difference $= d + 1 = 4$.

Total over 14 days:

$$S_{14} = \frac{14}{2}(2A + 13 \times 4) = 7(2A + 52) = 14A + 364 = 784$$

$$14A = 420 \implies A = 30$$

$A = \mathbf{30}$ minutes

End of Worked Solutions