Arithmetic Series Questions Sheet 2 Mark Scheme

Q1.

Question Number	Scheme		Marks
(a)	$206 = 140 + (12-1) \times d \Rightarrow d = ...$	Uses $206 = 140 + (12-1) \times d$ and proceeds as far as $d = ...$	M1
	$(d =) 6$	Correct answer only can score both marks.	A1
			(2)
(b)	$S_{12} = \frac{12}{2}(140 + 206)$ or $S_{12} = \frac{12}{2}(2 \times 140 + (12-1) \times "6")$ or $S_{11} = \frac{11}{2}(140 + 206 - "6")$ or $S_{11} = \frac{11}{2}(2 \times 140 + (11-1) \times "6")$	Attempts $S_n = \frac{n}{2}(a + l)$ or $S_n = \frac{n}{2}(2a + (n-1)d)$ with $n = 12$, $a = 140, l = 206, d = '6'$ **WAY 1** Or Attempts $S_n = \frac{n}{2}(a + l)$ or $S_n = \frac{n}{2}(2a + (n-1)d)$ with $n = 11$, $a = 140, l = 206 - '6', d = '6'$ **WAY2** If they are using $S_n = \frac{n}{2}(2a + (n-1)d)$, the n must be used consistently.	M1
	$S = 2076$ **WAY1** or $S = 1870$ **WAY 2**	Correct sum (may be implied)	A1
	$(52-12) \times 206 = ...$ or $(52-11) \times 206 = ...$	Attempts to find $(52-12) \times 206$ or $(52-11) \times 206$. Does **not** have to be consistent with their n used for the first Method mark.	M1
	Total $= "2076" + "8240" = ...$ **(WAY 1)** or Total $= "1870" + "8446" = ...$ **(WAY 2)**	Attempts to find the total by adding the sum to 12 terms with (52 - 12) lots of 206 or attempts to find the total by adding the sum to 11 terms with (52 - 11) lots of 206. I.e. consistency is now required for this mark. **Dependent on both previous method marks.**	**dd**M1
	10316	cao	A1
			(5)
			(7 marks)

Q2.

Question Number	Scheme	Marks
	(a) Use n^{th} term $= a+(n-1)d$ with $d = 10$; $a = 150$ and $n = 8$, or $a = 160$ and $n = 7$, or $a = 170$ and $n = 6$: $= 150+7\times10$ or $160+6\times10$ or $170+5\times10$	M1
	$= 220$* (Or gives clear list – see note)	A1* (2)
Or	If answer 220 is assumed and $150 + (n - 1)\ 10 = 220$ or variation is solved for $n=$	M1
	Then $n = 8$, so 2007 is the year (must conclude the year)	A1* (2)
	(b) Use $S_n = \frac{n}{2}\{2a+(n-1)10\}$ | Or $S_n = \frac{n}{2}\{a+l\}$ **and** $l = a + (n-1)10$	M1
	$= 7(300+13\times10)$ or $7(150 + 280)$	A1
	$= 7\times430$	
	$= 3010$	A1 (3)
	(c) Cost in year n $= 900+(n-1)\times-20$	M1
	Sales in year $n = 150+(n-1)\times10$	
	Cost $=3\times$Sales $\Rightarrow 900+(n-1)\times-20 = 3\times(150+(n-1)\times10)$	M1
	$900-20n+20 = 450+30n-30$	
	$500 = 50n$	
	$n = 10$	M1
	Year is 2009	A1
	As n is not defined they may work correctly from another base year to get the answer 2009 and their n may not equal 10. If doubtful – send to review.	(4)
		(9 marks)

Q3.

Question Number	Scheme	Marks
	Lewis; arithmetic series, $a = 140$, $d = 20$.	
(a)	$T_{20} = 140 + (20-1)(20); = 520$ Or lists 20 terms to get to 520	M1; A1
	OR $120 + (20)(20)$	[2]
	Method 1 / Method 2	
(b)	**Either:** Uses $\frac{1}{2}n(2a+(n-1)d)$ / **Or:** Uses $\frac{1}{2}n(a+l)$	M1
	$\frac{20}{2}(2\times140+(20-1)(20))$ / $\frac{20}{2}(140+\text{"520"})$ ft 520	A1
	6600	A1
		[3]
(c)	Sian; arithmetic series, $a = 300$, $l = 700$, $S_n = 8500$	
	Either: Attempt to use $8500 = \frac{n}{2}(a+l)$ / **Or:** May use both $8500 = \frac{1}{2}n(2a+(n-1)d)$ and $l = a+(n-1)d$ **and eliminate** d	M1
	$8500 = \frac{n}{2}(300+700)$ / $8500 = \frac{n}{2}(600+400)$	A1
	$\Rightarrow n = 17$	A1
		[3]
		8 marks

	Notes
(a)	**M1:** Attempt to use formula for **20**[th] **term** of Arithmetic series with first term **140** and d = **20. Normal formula rules** apply – see General principles at the start of the mark scheme re "Method Marks" **Or:** uses $120 + 20n$ with $n = 20$ **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520. M1A1 if correct M0A0 if wrong. (So 2 marks or zero) **A1:** For 520
(b)	**M1:** An attempt to apply $\frac{1}{2}n(2a+(n-1)d)$ **or** $\frac{1}{2}n(a+l)$ with their values for a, n, d and l **A1:** Uses $a = 140$, $d = 20$, $n = 20$ in their formula (two alternatives given above) but **ft on their value of** l from (a) if they use Method 2. **A1:** 6600 cao **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520 and adds **6600** gets M1A1A1- any other answer gets M1 A0A0 provided there are 20 numbers, the first is 140 and the last is 520.
(c) **First method**	**M1:** Attempt to use $S_n = \frac{n}{2}(a+l)$ with their values for a, and l and S =8500 **A1:** Uses formula with correct values **A1:** Finds exact value 17
Alternative method	**M1:** If both formulae $8500 = \frac{1}{2}n(2a+(n-1)d)$ and $l = a+(n-1)d$ are used, then d **must be eliminated** before this mark is awarded by valid work. Should not be using $d = 400$. This would be **M0**. **A1:** Correct equation in n only then **A1** for 17 exactly **Trial and error methods:** Finds $d = 25$ and $n = 17$ and list from 300 to 700 with total checked – 3/3

Q4.

Question Number	Scheme	Marks
	Boy's Sequence: 10, 15, 20, 25, ...	
(a)	$\{a=10,\ d=5 \Rightarrow T_{15}=\}\ a+14d = 10+14(5);\ = 80$ or $0.1+14(0.05);\ = £0.80$	M1; A1
		[2]
(b)	$\{S_{60}=\}\ \frac{60}{2}[2(10)+59(5)]$	M1 A1
	$= 30(315) = 9450$ or £94.50	A1
		[3]
	Boy's Sister's Sequence: 10, 20, 30, 40, ...	
(c)	$\{a=10,\ d=10 \Rightarrow S_m=\}\ \frac{m}{2}(2(10)+(m-1)(10))$ $\left(\text{or } \frac{m}{2}\times 10(m+1) \text{ or } 5m(m+1)\right)$	M1 A1
	63 or $6300 = \frac{m}{2}(2(10)+(m-1)(10))$	dM1
	$6300 = \frac{m}{2}(10)(m+1)$ or $12600 = 10m(m+1)$	
	$1260 = m(m+1)$	
	$35\times 36 = m(m+1)$ (*)	A1 cso
		[4]
(d)	$\{m=\}\ 35$	B1
		[1]
		10

Q5.

Question Number	Scheme	Notes	Marks
(a)	John; arithmetic series, $a = 60$, $d = 15$.		
	$60 + 75 + 90 = 225^*$ or $S_3 = \frac{3}{2}(120 + (3-1)(15)) = 225^*$	Finds and adds the first 3 terms or uses sum of 3 terms of an AP and obtains the printed answer, with no errors.	B1 *
	Beware: **The 12th term of the sequence is 225 also so look out for** $60+(12-1)\times 15 = 225$. **This is B0.**		
			[1]
(b)	$t_9 = 60 + (n-1)15 = (£)180$	M1: Uses $60 + (n-1)15$ with $n = 8$ or 9 A1: (£)180	M1 A1
	Listing: M1: Uses $a = 60$ and $d = 15$ to select the 8th or 9th term (allow arithmetic slips) A1: (£)180 (Special case (£)165 **only** scores M1A0)		
			[2]
(c)	$S_n = \frac{n}{2}(120 + (n-1)(15))$ or $S_n = \frac{n}{2}(60 + 60 + (n-1)(15))$	Uses correct formula for sum of n terms with $a = 60$ and $d = 15$ (must be a correct formula but ignore the value they use for n or could be in terms of n)	M1
	$S_n = \frac{12}{2}(120 + (12-1)(15))$	Correct numerical expression	A1
	$= (£)1710$	cao	A1
	Listing: M1: Uses $a = 60$ and $d = 15$ and finds the sum of at least 12 terms (allow arithmetic slips) A2: (£)1710		
			[3]
(d)	$3375 = \frac{n}{2}(120 + (n-1)(15))$	Uses correct formula for sum of n terms with $a = 60$, $d = 15$ and puts $= 3375$	M1
	$6750 = 15n(8 + (n-1)) \Rightarrow 15n^2 + 105n = 6750$	**Correct three term** quadratic. E.g. $6750 = 105n + 15n^2$, $3375 = \frac{15}{2}n^2 + \frac{105}{2}n$ This may be implied by equations such as $6750 = 15n(n+7)$ or $3375 = \frac{15}{2}(n^2 + 7n)$	A1
	$n^2 + 7n = 25 \times 18$ *	Achieves the printed answer with no errors but must see the 450 or 450 in factorised form or e.g. 6750, 3375 in factorised form i.e. an intermediate step.	A1*
			[3]
(e)	$n = 18 \Rightarrow$ Aged 27	M1: Attempts to solve the given quadratic or states $n = 18$ A1: Age = 27 or just 27	M1 A1
	Age = 27 only scores both marks (i.e. $n = 18$ need not be seen)		
	Note that (e) is not hence so allow valid attempts to solve the given equation for M1		
			[2]
			11 marks

Q6.

Question	Scheme	Marks	AOs
(a)	$16+(21-1)\times d=24 \Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6 \Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57900$	A1	1.1b
		(4 marks)	

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using $a = 16$, and $n = 21$ and the 24. The method may be implied by their working.
If the AP term formula is quoted it must be correct, so use of e.g. $u_n = a + nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with $a = 16$, $n = 500$ and their numerical d from part (a)
If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n = a + nd$ by showing the following:

(a) $d=\frac{24-16}{21}=\frac{8}{21}$ (b) $S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$

This scores (a) M0A0 (b) M1A0

Q7.

Question Number	Scheme	Marks
(*a*)	5, 7, 9, 11 or $5+2+2+2=11$ or $5+6=11$	
	use $a=5$, $d=2$, $n=4$ and $t_4=5+3\times2=11$	B1 (1)
(*b*)	$t_n=a+(n-1)d$ with one of $a=5$ or $d=2$ correct	M1
	$=5+2(n-1)$ or $2n+3$ or $1+2(n+1)$	A1 (2)
(*c*)	$S_n=\frac{n}{2}[2\times5+2(n-1)]$ or use of $\frac{n}{2}(5+\text{"their } 2n+3\text{"})$	M1 A1
	$=\{n(5+n-1)\}=n(n+4)$ (*)	A1 cso (3)
(*d*)	$43 = 2n+3$	M1
	$[n]=20$	A1 (2)
(*e*)	$S_{20}=20\times24$, $=\underline{480}$ (km)	M1 A1 (2)
		(10 marks)

Q8.

Question Number	Scheme		Marks
(a)	$32000 = 17000 + (k-1)\times 1500 \Rightarrow k = ...$	Use of 32000 with a correct formula in an attempt to find k. A correct formula could be implied by a correct answer.	M1
	$(k=)$ 11	Cso (Allow $n = 11$)	A1
	Accept correct answer only.		
	$32000 = 17000 + 1500k \Rightarrow k = 10$ is M0A0 (wrong formula) $\frac{32000-17000}{1500} = 10 \therefore k = 11$ is M1A1 (correct formula implied)		
	Listing: All terms must be listed up to 32000 and 11 correctly identified. A solution that scores 2 if fully correct and 0 otherwise.		
			(2)
(b)	M1: $S = \frac{k}{2}(2\times 17000 + (k-1)\times 1500)$ or $\frac{k}{2}(17000 + 32000)$ $S = \frac{k-1}{2}(2\times 17000 + (k-2)\times 1500)$ or $\frac{k-1}{2}(17000 + 30500)$ A1: $S = \frac{11}{2}(2\times 17000 + 10\times 1500)$ or $\frac{11}{2}(17000 + 32000)$ $S = \frac{10}{2}(2\times 17000 + 9\times 1500)$ or $\frac{10}{2}(17000 + 30500)$ (= 269 500 or 237 500)	M1: Use of correct sum formula with their **integer** $n = k$ or $k - 1$ from part (a) where $3 < k < 20$ and a = 17000 and d = 1500. **See below for special case for using $n = 20$.** A1: Any correct un-simplified numerical expression with $n = 11$ or $n = 10$	M1A1
	$32000\times\alpha$	$32000\times\alpha$ where α is an integer and $3 < \alpha < 18$	M1
	288 000 + 269 500 = 557 500 or 320 000 + 237 500 = 557 500	M1: Attempts to add their two values. It is dependent upon the two previous M's being scored and must be the sum of 20 terms i.e. $\alpha + k = 20$ A1: 557 500	ddM1A1
	Special Case: If they just find S_{20} (£625 000) in (b) score the first M1 otherwise apply the scheme.		
			(5)
			(7 marks)

Listing:

n	1	2	3	4	5	6	7	8	9	10
u_n	17000	18500	20000	21500	23000	24500	26000	27500	29000	30500
n	11	12	13	14	15	16	17	18	19	20
u_n	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000

Look for a sum before awarding marks. Award the M's as above then A2 for 557 500
If they sum the 'parts' separately then apply the scheme.

Q9.

Question Number	Scheme	Marks
(a).	Attempts to use $a+(n-1)"d"$ with $a=A$ and "d"$=d+1$ and $n=14$	M1
	$A+13(d+1)=A+13d+13$ *	A1*
		(2)
(b)	Calculates time for Yi on Day 14$=(A-13)+13(2d-1)$	M1
	Sets **times** equal $A+13d+13=\ (A-13)+13(2d-1)\Rightarrow d=...$	M1
	$d=3$	A1 cso
		(3)
(c)	Uses $\frac{n}{2}\{2A+(n-1)(D)\}$ with $n=14$, and with $D=d$ or $d+1$	M1
	Attempts to solve $\frac{14}{2}\{2A+13\times'(d+1)'\}=784\Rightarrow A=...$	dM1
	$A=30$	A1
		(3)
		(8 marks)

(a) M1 Attempts to use $a+(n-1)d$ with $a=A$ and $d=d+1$ AND $n=14$

A1* cao This is a given answer and there is an expectation that the intermediate answer is seen and that **all work is correct** with correct brackets.

The expressions $A+13(d+1)$ and $A+13d+13$ should be seen

N.B. If **brackets are missing and formula is not stated**

e.g. $A+13d+1\Rightarrow A+13d+13$ or $A+(13)d+1\Rightarrow A+13d+13$ then this is **M0A0**

If formula is quoted and $a=A$ and $d=d+1$ is quoted or implied, then M1 A0 may be given

So $a+(n-1)d$ followed by $A+(13)d+1=A+13d+13$ achieves **M1A0**

(b) M1 States a time for Yi on Day 14 = $(A-13)+13(2d-1)$

M1 Sets their **time** for Yi, equal to $A+13d+13$ and uses this equation to proceed to $d=$

A1 cso $d=3$ Needs both M marks and must be simplified to 3 (not 39/13)

[NB Setting **each** of the times separately equal to 0 leads to $d=3$ – this will gain M0A0]

(c) M1 Uses the sum formula $\frac{n}{2}\{2A+(n-1)(D)\}$ with $n=14$ and $D=d+1$ or allow $D=d$ (usually 4 or 3)

NB May use $\frac{n}{2}\{A+(A+13D)\}$ with $n=14$ and and $D=d+1$ or allow $D=d$ (usually 4 or 3)

dM1 Attempts to solve $\frac{14}{2}\{2A+13\times'4'\}="784"\Rightarrow A=...$ (Must use their $d+1$ this time)

Allow miscopy of 784

A1 cao $A=30$