Question 1 (OCR 4722, Jun 2008, Q5)

Worked Solution

Part (i): Show the shaded area equals $\int_5^7 (y^2 - 6y + 7)\,dy$

The curve is $y = 3 + \sqrt{x+2}$, so we need to express x in terms of y.

Starting from $y = 3 + \sqrt{x+2}$:

$$y - 3 = \sqrt{x+2} \implies (y-3)^2 = x + 2 \implies x = (y-3)^2 - 2 = y^2 - 6y + 9 - 2 = y^2 - 6y + 7$$

The required area is $\int x\,dy$.

Finding the y-limits: when $x = 2$, $y = 3 + \sqrt{2+2} = 3 + 2 = 5$; when $x = 14$, $y = 3 + \sqrt{14+2} = 3 + 4 = 7$.

Therefore the shaded area $= \int_5^7 (y^2 - 6y + 7)\,dy$. **(shown)**

Part (ii): Find the exact area

$$\int_5^7 (y^2 - 6y + 7)\,dy = \left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7$$

Evaluating at $y = 7$: $\frac{343}{3} - 147 + 49 = \frac{343}{3} - 98$

Evaluating at $y = 5$: $\frac{125}{3} - 75 + 35 = \frac{125}{3} - 40$

Subtracting:

$$\left(\frac{343}{3} - 98\right) - \left(\frac{125}{3} - 40\right) = \frac{343 - 125}{3} - 58 = \frac{218}{3} - 58 = \frac{218 - 174}{3} = \frac{44}{3} = 16\tfrac{2}{3} - 2 = 14\tfrac{2}{3}$$

Let us be careful:

$$= \frac{343}{3} - 147 + 49 - \frac{125}{3} + 75 - 35 = \frac{218}{3} - 58 = \frac{218 - 174}{3} = \frac{44}{3}$$

Exact area $= \frac{44}{3} = 14\tfrac{2}{3}$

Question 2 (OCR 4722, Jun 2011, Q4)

Worked Solution

Part (i): Show $y = -1 + \sqrt{x+4}$ rearranges to $x = y^2 + 2y - 3$

Starting from $y = -1 + \sqrt{x+4}$:

$$y + 1 = \sqrt{x+4} \implies (y+1)^2 = x + 4 \implies y^2 + 2y + 1 = x + 4 \implies x = y^2 + 2y - 3$$

$x = y^2 + 2y - 3$ **(shown)**

Part (ii): Find the exact shaded area

The shaded region is bounded by the curve, the y-axis ($x = 0$), and the line $y = 3$.

First, find the lower y-limit by setting $x = 0$:

$$0 = y^2 + 2y - 3 = (y+3)(y-1) \implies y = 1 \quad \text{(taking positive root in context)}$$

So the area $= \displaystyle\int_1^3 (y^2 + 2y - 3)\, dy$:

$$= \left[\frac{1}{3}y^3 + y^2 - 3y\right]_1^3$$

At $y = 3$: $9 + 9 - 9 = 9$

At $y = 1$: $\dfrac{1}{3} + 1 - 3 = \dfrac{1}{3} - 2 = -\dfrac{5}{3}$

$$\text{Area} = 9 - \left(-\frac{5}{3}\right) = 9 + \frac{5}{3} = \frac{27+5}{3} = \frac{32}{3} = 10\tfrac{2}{3}$$

Exact area $= \dfrac{32}{3} = 10\tfrac{2}{3}$

Question 3 (OCR 4722, Jun 2014, Q9)

Worked Solution

The curve is $y = -3 + 2\sqrt{x+4}$. Region A is bounded by the curve, the x-axis, the y-axis and $x = 5$. Region B is bounded by the curve, the y-axis and $y = 3$.

Part (i): Trapezium rule with 2 strips of width 2.5 for region A

The trapezium rule with $h = 2.5$ and x-values $0, 2.5, 5$:

y-values:

- $x = 0$: $y = -3 + 2\sqrt{4} = -3 + 4 = 1$
- $x = 2.5$: $y = -3 + 2\sqrt{6.5} \approx -3 + 2(2.5495) \approx 2.099$
- $x = 5$: $y = -3 + 2\sqrt{9} = -3 + 6 = 3$

Trapezium rule:

$$\text{Area} \approx \frac{h}{2}(y_0+2y_1+y_2) = \frac{2.5}{2}(1+2(2.099)+3) = 1.25\times(1+4.198+3) = 1.25\times 8.198 \approx 10.2$$

Approximate area of region $A \approx$ **10.2** (3 s.f.)

Part (ii): Approximate area of region B

Region B is within the rectangle of width 5 and height 3 (from $x = 0$ to $x = 5$, $y = 0$ to $y = 3$).

Area of rectangle $= 5 \times 3 = 15$.

Region A + Region B = area of rectangle (since $P(5, 3)$ lies on the curve):

$$\text{Area of } B \approx 15 - 10.2 = 4.8$$

Approximate area of region $B \approx$ **4.8**

Part (iii): Show by integration that exact area of region B is $\frac{14}{3}$

First write $x = f(y)$. From $y = -3 + 2\sqrt{x+4}$:

$$y+3 = 2\sqrt{x+4} \implies \frac{(y+3)^2}{4} = x+4 \implies x = \frac{(y+3)^2}{4} - 4 = \frac{y^2+6y+9}{4} - 4 = \frac{y^2+6y-7}{4}$$

The y-limits for region B: when $x = 0$: $\frac{y^2+6y-7}{4} = 0 \Rightarrow (y+7)(y-1) = 0 \Rightarrow y = 1$; upper limit is $y = 3$.

$$\text{Area of } B = \int_1^3 \frac{y^2+6y-7}{4}\,dy = \frac{1}{4}\left[\frac{y^3}{3} + 3y^2 - 7y\right]_1^3$$

At $y = 3$: $\dfrac{27}{3} + 27 - 21 = 9 + 27 - 21 = 15$

At $y = 1$: $\dfrac{1}{3} + 3 - 7 = \dfrac{1}{3} - 4 = -\dfrac{11}{3}$

$$\text{Area} = \frac{1}{4}\left(15 - \left(-\frac{11}{3}\right)\right) = \frac{1}{4}\left(\frac{45}{3} + \frac{11}{3}\right) = \frac{1}{4} \times \frac{56}{3} = \frac{56}{12} = \frac{14}{3}$$

Exact area of region $B = \dfrac{14}{3}$ **(shown)**

End of Worked Solutions