



Applied Sequences and Series Exam Questions (From OCR MEI 4752)

Q1, (Jun 2010, Q12)

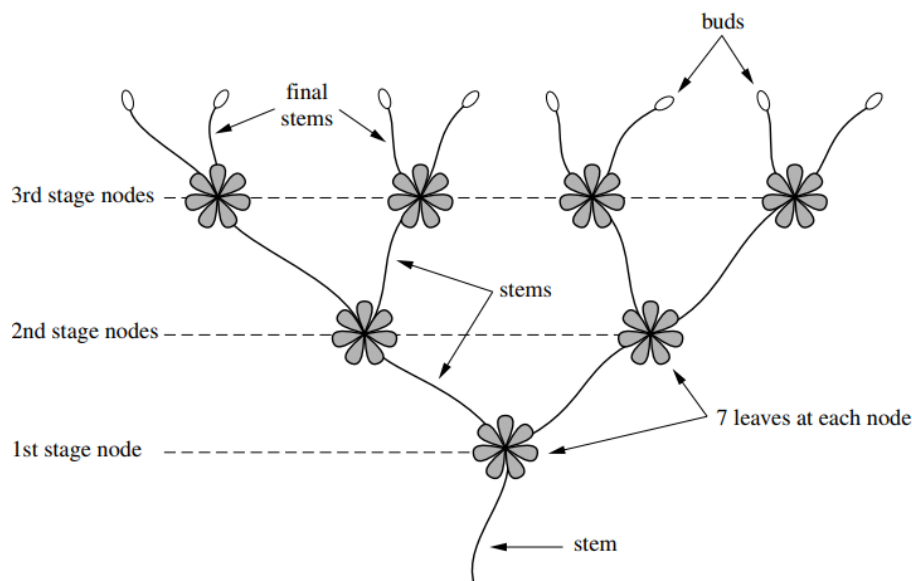


Fig. 12

A branching plant has stems, nodes, leaves and buds.

- There are 7 leaves at each node.
- From each node, 2 new stems grow.
- At the end of each final stem, there is a bud.

Fig. 12 shows one such plant with 3 stages of nodes. It has 15 stems, 7 nodes, 49 leaves and 8 buds.

(i) One of these plants has 10 stages of nodes.

(A) How many buds does it have? [2]

(B) How many stems does it have? [2]

(ii) (A) Show that the number of leaves on one of these plants with n stages of nodes is

$$7(2^n - 1). \quad [2]$$

(B) One of these plants has n stages of nodes and more than 200 000 leaves. Show that n satisfies the inequality $n > \frac{\log_{10} 200\,007 - \log_{10} 7}{\log_{10} 2}$. Hence find the least possible value of n .

[4]



Q2, (Jun 2005, Q11)

There is a flowerhead at the end of each stem of an oleander plant. The next year, each flowerhead is replaced by three stems and flowerheads, as shown in Fig. 11.

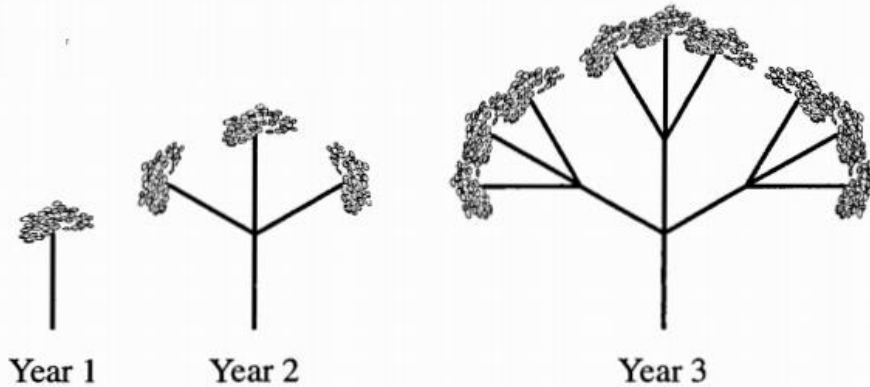


Fig. 11

- (i) How many flowerheads are there in year 5? [1]
- (ii) How many flowerheads are there in year n ? [1]
- (iii) As shown in Fig. 11, the total number of stems in year 2 is 4, (that is, 1 old one and 3 new ones). Similarly, the total number of stems in year 3 is 13, (that is, 1 + 3 + 9).

Show that the total number of stems in year n is given by $\frac{3^n - 1}{2}$. [2]

- (iv) Kitty's oleander has a total of 364 stems. Find

(A) its age, [2]

(B) how many flowerheads it has. [1]

- (v) Abdul's oleander has over 900 flowerheads.

Show that its age, y years, satisfies the inequality $y > \frac{\log_{10} 900}{\log_{10} 3} + 1$.

Find the smallest integer value of y for which this is true. [4]



Q3, (Jun 2007, Q11)

(a) André is playing a game where he makes piles of counters. He puts 3 counters in the first pile. Each successive pile he makes has 2 more counters in it than the previous one.

(i) How many counters are there in his sixth pile? [1]

(ii) André makes ten piles of counters. How many counters has he used altogether? [2]

(b) In another game, played with an ordinary fair die and counters, Betty needs to throw a six to start.

The probability P_n of Betty starting on her n th throw is given by

$$P_n = \frac{1}{6} \times \left(\frac{5}{6}\right)^{n-1}.$$

(i) Calculate P_4 . Give your answer as a fraction. [2]

(ii) The values P_1, P_2, P_3, \dots form an infinite geometric progression. State the first term and the common ratio of this progression.

Hence show that $P_1 + P_2 + P_3 + \dots = 1$. [3]

(iii) Given that $P_n < 0.001$, show that n satisfies the inequality

$$n > \frac{\log_{10} 0.006}{\log_{10} \left(\frac{5}{6}\right)} + 1.$$

Hence find the least value of n for which $P_n < 0.001$. [4]



Q4, (Jun 2011, Q12)

Jim and Mary are each planning monthly repayments for money they want to borrow.

- (i) Jim's first payment is £500, and he plans to pay £10 less each month, so that his second payment is £490, his third is £480, and so on.
- (A) Calculate his 12th payment. [2]
- (B) He plans to make 24 payments altogether. Show that he pays £9240 in total. [2]
- (ii) Mary's first payment is £460 and she plans to pay 2% less each month than the previous month, so that her second payment is £450.80, her third is £441.784, and so on.
- (A) Calculate her 12th payment. [2]
- (B) Show that Jim's 20th payment is less than Mary's 20th payment but that his 19th is not less than her 19th. [3]
- (C) Mary plans to make 24 payments altogether. Calculate how much she pays in total. [2]
- (D) How much would Mary's first payment need to be if she wishes to pay 2% less each month as before, but to pay the same in total as Jim, £9240, over the 24 months? [2]

Q5, (Jun 2009, Q11)

- (i) In a 'Make Ten' quiz game, contestants get £10 for answering the first question correctly, then a further £20 for the second question, then a further £30 for the third, and so on, until they get a question wrong and are out of the game.
- (A) Haroon answers six questions correctly. Show that he receives a total of £210. [1]
- (B) State, in a simple form, a formula for the total amount received by a contestant who answers n questions correctly.
- Hence find the value of n for a contestant who receives £10 350 from this game. [4]
- (ii) In a 'Double Your Money' quiz game, contestants get £5 for answering the first question correctly, then a further £10 for the second question, then a further £20 for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.
- (A) Gary received £75 from the game. How many questions did he get right? [1]
- (B) Bethan answered 9 questions correctly. How much did she receive from the game? [2]
- (C) State a formula for the total amount received by a contestant who answers n questions correctly.
- Hence find the value of n for a contestant in this game who receives £2 621 435. [4]
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Q6. (Jun 2015, Q11)

Jill has 3 daughters and no sons. They are generation 1 of Jill's descendants.

Each of her daughters has 3 daughters and no sons. Jill's 9 granddaughters are generation 2 of her descendants. Each of her granddaughters has 3 daughters and no sons; they are descendant generation 3.

Jill decides to investigate what would happen if this pattern continues, with each descendant having 3 daughters and no sons.

- (i) How many of Jill's descendants would there be in generation 8? [2]
- (ii) How many of Jill's descendants would there be altogether in the first 15 generations? [3]
- (iii) After n generations, Jill would have over a million descendants altogether. Show that n satisfies the inequality

$$n > \frac{\log_{10} 2000003}{\log_{10} 3} - 1.$$

Hence find the least possible value of n . [4]

- (iv) How many **fewer** descendants would Jill have altogether in 15 generations if instead of having 3 daughters, she and each subsequent descendant has 2 daughters? [3]
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