Question 1

Worked Solution

(a) The sequence is arithmetic with $a = 140$, and the 12th term is 206. Using $u_n = a + (n-1)d$:

$$206 = 140 + (12-1) \times d$$

$$66 = 11d$$

$d = 6$

(b) Step 1: Find the sum of the first 12 weeks (arithmetic part).

Using $S_n = \frac{n}{2}(a + l)$ with $n = 12$, $a = 140$, $l = 206$:

$$S_{12} = \frac{12}{2}(140 + 206) = 6 \times 346 = 2076$$

Step 2: Find the number of weeks at constant production.

After week 12, the company makes 206 bicycles per week for the remaining $52 - 12 = 40$ weeks.

Step 3: Find the total for weeks 13–52.

$$40 \times 206 = 8240$$

Step 4: Add to find the total.

$$\text{Total} = 2076 + 8240 = 10316$$

Total number of bicycles $= 10316$

Question 2

Worked Solution

(a) For the first 4 km she runs at 6 min/km, so time $= 4 \times 6 = 24$ minutes.

The 5th km takes 6×1.05 minutes, and the 6th km takes 6×1.05^2 minutes.

Total time for 6 km:

$$24 + 6 \times 1.05 + 6 \times 1.05^2 = 24 + 6.3 + 6.615 = 36.915 \text{ minutes}$$

Converting: $0.915 \times 60 = 54.9 \approx 55$ seconds.

Time for first 6 km = 36 minutes 55 seconds ✓

(b) The 5th km takes 6×1.05^1 minutes, the 6th km takes 6×1.05^2 minutes, the 7th km takes 6×1.05^3 minutes, ...

In general, the rth km (for $5 \le r \le 20$) takes:

$6 \times 1.05^{r-4}$ minutes ✓

(c) Step 1: Write the total time as the first 4 km plus the sum from km 5 to km 20.

$$\text{Total} = 24 + \sum_{r=5}^{20} 6 \times 1.05^{r-4}$$

This geometric sum has first term $a = 6 \times 1.05^1 = 6.3$, common ratio $r = 1.05$, and $n = 20 - 5 + 1 = 16$ terms.

Step 2: Apply the geometric sum formula.

$$\text{Total} = 24 + 6.3 \times \frac{1.05^{16} - 1}{1.05 - 1} = 24 + 6.3 \times \frac{1.05^{16} - 1}{0.05}$$

$$= 24 + 149.04\ldots = 173.04\ldots \text{ minutes}$$

Step 3: Convert to minutes and seconds.

$$0.04\ldots \times 60 \approx 3 \text{ seconds}$$

Total time $\approx$ 173 minutes and 3 seconds

Question 3

Worked Solution

(a)(i) We are given $V = Ap^t$, with $V = 32000$ when $t = 4$ (year 2005) and $V = 50000$ when $t = 11$ (year 2012).

Step 1: Divide to eliminate A.

$$\frac{50000}{32000} = \frac{Ap^{11}}{Ap^4} = p^7$$

$$p^7 = \frac{50000}{32000} = \frac{25}{16}$$

$$p = \left(\frac{25}{16}\right)^{1/7}$$

$p = 1.0658$ (to 4 d.p.)

(a)(ii) Substituting $p = 1.0658$ and $t = 4$, $V = 32000$:

$$A = \frac{32000}{1.0658^4} = \frac{32000}{1.2904\ldots} \approx 24795 \approx 24800 \quad \checkmark$$

(b)(i) $A \approx 24800$ is the value of the car (in £) on 1st January 2001, when $t = 0$.

(b)(ii) $p \approx 1.0658$ is the factor by which the value increases each year; equivalently the car's value rises by approximately 6.6% per year.

(c) We require the year when $V > 100000$:

$$24800 \times 1.0658^t = 100000$$

$$1.0658^t = \frac{100000}{24800}$$

Taking logarithms:

$$t = \log_{1.0658}\left(\frac{100000}{24800}\right) = \frac{\ln\left(\frac{100000}{24800}\right)}{\ln(1.0658)}$$

$$t \approx 21.8$$

Since t is measured from 1st January 2001, the value first exceeds £100 000 during the year $2001 + 22 = 2023$. However, checking: at $t = 21$ (start of 2022) $V < 100000$, and at $t = 22$ (start of 2023) $V > 100000$.

The value first exceeds £100 000 during the year **2022**.

Note: the mark scheme gives 2022, since $t \approx 21.8$ means the threshold is crossed during the 22nd year from 2001, i.e. during 2022.

Question 4

Worked Solution

(a) The arithmetic sequence has $a = 17000$, $d = 1500$, and the kth term equals 32000.

$$32000 = 17000 + (k-1) \times 1500$$

$$15000 = (k-1) \times 1500$$

$$k - 1 = 10 \implies k = 11$$

$k = 11$

(b) Step 1: Sum of years 1 to 11 (arithmetic part).

Using $S_n = \frac{n}{2}(a+l)$ with $n = 11$, $a = 17000$, $l = 32000$:

$$S_{11} = \frac{11}{2}(17000 + 32000) = \frac{11}{2} \times 49000 = 269500$$

Step 2: Sum of years 12 to 20 (constant at £32 000).

Number of years at £32 000: $20 - 11 = 9$ years.

$$9 \times 32000 = 288000$$

Step 3: Total earnings.

$$\text{Total} = 269500 + 288000 = 557500$$

Total earnings over 20 years = £557 500

Question 5

Worked Solution

(a) Arithmetic sequence model.

The sequence has $a = 28$, and the 6th term is 115. Using $u_n = a + (n-1)d$:

$$115 = 28 + 5d \implies d = \frac{87}{5} = 17.4$$

The 3rd gear speed:

$$u_3 = 28 + 2 \times 17.4 = 28 + 34.8 = 62.8$$

Fastest speed in 3rd gear = 62.8 km h^{-1}

(b) Geometric sequence model.

The sequence has $a = 28$, and the 6th term is 115. Using $u_n = ar^{n-1}$:

$$115 = 28 \times r^5 \implies r^5 = \frac{115}{28} \implies r = \left(\frac{115}{28}\right)^{1/5} \approx 1.3265$$

The 5th gear speed:

$$u_5 = 28 \times 1.3265^4 = 28 \times \frac{115}{28 \times 1.3265} = \frac{115}{1.3265} \approx 86.7$$

Fastest speed in 5th gear ≈ 86.7 km h^{-1}

End of Worked Solutions