

**Variable Acceleration in Two Dimensions (From OCR 4761)**

**Q1, (Jan 2005, Q1)**

(i)	Differentiate $\mathbf{v} = 2t \mathbf{i} + (5 - 4t) \mathbf{j}$	M1 A1	At least 1 cpt correct Award for RHS seen	4
	Differentiate $\mathbf{a} = 2 \mathbf{i} - 4 \mathbf{j}$	M1 F1	Do not award if $\mathbf{i}$ and $\mathbf{j}$ lost in $\mathbf{v}$ . At least 1 cpt correct. FT FT from <b>their</b> 2 component $\mathbf{v}$	
(ii)	$\mathbf{F} + 12 \mathbf{j} = 4(2 \mathbf{i} - 4 \mathbf{j})$	M1	N2L. Allow $\mathbf{F} = mg \mathbf{a}$ . No extra forces. Allow 12j omitted	3
		A1	Allow wrong signs otherwise correct with <b>their</b> vector $\mathbf{a}$ .	
	$\mathbf{F} = 8 \mathbf{i} + 28 \mathbf{j}$	A1	cao	
	total	7		

**Q2, (Jun 2005, Q5)**

(i)	$x = 2 \Rightarrow t = 4$ $t = 4 \Rightarrow y = 16 - 1 = 15$	B1 F1	cao FT <b>their</b> $t$ and $y$ . Accept 15 j	2
(ii)	$x = \frac{1}{2}t$ and $y = t^2 - 1$	M1	Attempt at elimination of expressions for $x$ and $y$ in terms of $t$	2
	Eliminating $t$ gives $y = ((2x)^2 - 1) = 4x^2 - 1$	E1	Accept seeing $(2x)^2 - 1 = 4x^2 - 1$	
(iii)	<b>either</b>			3
	We require $\frac{dy}{dx} = 1$	M1	This may be implied	
	so $8x = 1$	B1	Differentiating correctly to obtain $8x$	
	$x = \frac{1}{8}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	A1		
	<b>or</b>			
Differentiate to find $\mathbf{v}$ equate $\mathbf{i}$ and $\mathbf{j}$ cpts	M1 M1	Equating the $\mathbf{i}$ and $\mathbf{j}$ cpts of <b>their</b> $\mathbf{v}$		
so $t = \frac{1}{4}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	A1			
	total	7		

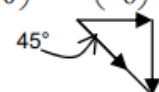
**Q3, (Jun 2006, Q4)**

<p>(i) <b>either</b>                  Need <b>j</b> cpt 0 so <math>18t^2 - 1 = 0</math>  <math>\Rightarrow t^2 = \frac{1}{18}</math>. Only one root as <math>t &gt; 0</math></p> <p><b>or</b>                  Establish sign change in <b>j</b> cpt                  Establish only one root</p>	<p>M1 Need not solve                  E1 Must establish only one of the two roots is valid</p> <p>B1                  B1</p>	<p>2</p>
<p>(ii) <math>\mathbf{v} = 3\mathbf{i} + 36t\mathbf{j}</math>                   Need <b>i</b> cpt 0 and this never happens</p>	<p>M1 Differentiate. Allow <b>i</b> or <b>j</b> omitted                  A1                  E1 Clear explanation. Accept '<b>i</b> cpt always there' or equiv</p>	<p>3</p>
<p>(iii) <math>x = 3t</math> and <math>y = 18t^2 - 1</math>                  Eliminate <math>t</math> to give  <math>y = 18\left(\frac{x}{3}\right)^2 - 1</math>                  so <math>y = 2x^2 - 1</math></p>	<p>B1 Award for these two expressions seen.</p> <p>M1 <math>t</math> properly eliminated. Accept any form and brackets missing                  A1 cao</p>	<p>3                  8</p>

**Q4, (Jan 2006, Q5)**

(i)	$9\mathbf{i} \text{ m s}^{-2}; (9\mathbf{i} - 12\mathbf{j}) \text{ m s}^{-2}$	B1	Award for either. Accept no units. (isw e.g. finding magnitudes)	1
(ii)	N2L $\mathbf{F} = 4(9\mathbf{i} - 12\mathbf{j}) = (36\mathbf{i} - 48\mathbf{j}) \text{ N}$	B1	Accept factored form. isw. FT <b>a(3)</b> . Accept 60 N or <b>their</b> $4 a $	1
(iii)	$\mathbf{v} = \int \begin{pmatrix} 9 \\ -4t \end{pmatrix} dt = \begin{pmatrix} 9t + C \\ -2t^2 + D \end{pmatrix}$ <p>Using <math>\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}</math> when <math>t = 1</math></p> $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 + C \\ -2 + D \end{pmatrix}$ <p><math>\Rightarrow C = -5, D = 4</math> so <math>\mathbf{v} = (9t - 5)\mathbf{i} + (4 - 2t^2)\mathbf{j}</math></p>	<p>M1 Integration. At least one term correct.                  A1 Neglect arbitrary constant(s)                  M1 Sub at <math>t = 1</math> to find arb const(s)</p> <p>A1 Any form</p>	<p>4</p>	
				6

**Q5, (Jan 2007, Q6)**

<p>(i)</p> $t = 2.5 \Rightarrow \mathbf{v} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + 2.5 \begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$  <p>speed is <math>\sqrt{10^2 + 10^2} = 14.14\dots</math> so <math>14.1 \text{ m s}^{-1}</math> (3 s. f.)</p>	<p>B1 E1 F1</p>	<p>Need not be in vector form Accept diag and/or correct derivation of just <math>\pm 45^\circ</math> FT <b>their v</b></p>	<p>3</p>
<p>(ii)</p> $\mathbf{s} = 2.5 \begin{pmatrix} -5 \\ 10 \end{pmatrix} + \frac{1}{2} \times 2.5^2 \times \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ $= \begin{pmatrix} 6.25 \\ 0 \end{pmatrix}$ <p>so <math>090^\circ</math></p>	<p>M1 A1 A1 A1</p>	<p>Consideration of <b>s</b> (const accn or integration) Correct sub into <i>uvast</i> with <b>u</b> and <b>a</b>. (If integration used it must be correct but allow no arb constant) cao. CWO.</p>	<p>4 7</p>

**Q6, (Jun 2009, Q5)**

<p>(i)</p> $\mathbf{v} = \mathbf{i} + (3 - 2t)\mathbf{j}$ $\mathbf{v}(4) = \mathbf{i} - 5\mathbf{j}$	<p>M1 A1 F1</p>	<p>Differentiating <b>r</b>. Allow 1 error. Could use const accn. Do not award if <math>\sqrt{26}</math> is given as vel (accept if <b>v</b> given and <i>v</i> given as well called speed or magnitude).</p>	<p>3</p>
<p>(ii)</p> $\mathbf{a} = -2\mathbf{j}$ <p>Using N2L <math>\mathbf{F} = 1.5 \times (-2\mathbf{j})</math></p> <p>so <math>-3\mathbf{j} \text{ N}</math></p>	<p>B1 M1 A1</p>	<p>Diff <b>v</b>. FT <b>their v</b>. Award if <math>-2\mathbf{j}</math> seen &amp; isw. Award for <math>1.5 \times (\pm \text{their } \mathbf{a} \text{ or } a)</math> seen. cao Do not award if final answer is not correct. [Award M1 A1 for <math>-3\mathbf{j}</math> WW]</p>	<p>3</p>
<p>(iii)</p> $x = 2 + t \text{ and } y = 3t - t^2$ <p>Substitute <math>t = x - 2</math></p> $\text{so } y = 3(x - 2) - (x - 2)^2$ $[ = (x - 2)(5 - x) ]$	<p>B1 B1</p>	<p>Must have both but may be implied. cao. isw. Must see the form <math>y = \dots</math></p>	<p>2</p>

**Q7, (Jun 2013, Q4)**

Equate **i** and **j** components of **v**

$$16 - t^2 = 31 - 8t$$

$$t^2 - 8t + 15 = 0$$

$$(t - 3)(t - 5) = 0$$

$$t = 3 \text{ or } 5$$

When  $t = 3$ ,  $\mathbf{v} = 7\mathbf{i} + 7\mathbf{j}$

Speed when  $t = 3$  is  $7\sqrt{2} = 9.9 \text{ m s}^{-1}$

The values of the **i** and **j** components must both be positive for the bearing to be  $045^\circ$ .

M1 The candidate recognises that the **i** and **j** components must be equal.

A1 An equation is formed.

A1 May be implied by later working.

B1

B1

B1 This mark is dependent on obtaining A1 for the result  $t = 3$  or  $5$ . It is awarded if the speed for the case when  $t = 5$  is not included (since  $t = 5 \Rightarrow \mathbf{v} = -9\mathbf{i} - 9\mathbf{j}$  and the bearing is  $225^\circ$ ).

Note Candidates who obtain **r** and equate the east and north components should be awarded SC1 for the whole question.

[6]

**Q8, (Jan 2011, Q4)**

(i)	<p>When <math>t = 1</math>, <math>\mathbf{r} = \begin{pmatrix} 8 \\ 10-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}</math>  <math>[8\mathbf{i} + (10 - 2)\mathbf{j} = 8\mathbf{i} + 8\mathbf{j}]</math>                      Bearing OP is <math>045^\circ</math></p>	<p>B1 F1 2</p>	<p>Accept column or <math>a\mathbf{i} + b\mathbf{j}</math> notation                      May be implied                      Accept <math>45^\circ</math>. Accept NE and northeast. Condone <math> \mathbf{r} </math> given as well.</p>
(ii)	<p><math>\mathbf{v} = \begin{pmatrix} 8 \\ 20t - 6t^2 \end{pmatrix} [8\mathbf{i} + (20t - 6t^2)\mathbf{j}]</math>                      The <math>\mathbf{i}</math> cpt is always 8 so <math>\mathbf{v} \neq \mathbf{0}</math> for any <math>t</math></p>	<p>M1 A1 E1 3</p>	<p>Differentiating both components. Condone 1 error if clearly attempting differentiation.                      Must be a vector answer.                      Accept any correct argument e.g. based on <math>\mathbf{i}</math> cpt never 0.</p>
(iii)	<p><math>\mathbf{a} = \begin{pmatrix} 0 \\ 20-12t \end{pmatrix} [(20-12t)\mathbf{j}]</math>  <math>\mathbf{a} = \mathbf{0}</math> when <math>t = \frac{20}{12} = \frac{5}{3}</math>                      so <math>\frac{5}{3}</math> s (1.67 s (3 s. f.))</p>	<p>M1 F1 B1 3</p>	<p>Differentiating as a vector. Condone 1 error if clearly attempting differentiation of <b>their v</b>.                      FT <b>their v</b>.                      cao. Condone obtained from scalar equation.</p>
		<p>8</p>	

**Q9, (Jun 2015, Q3)**

<p><b>(i)</b></p>	<p><b>Either</b> <math>-2 + 8t = 7t</math>  <b>Or</b> <math>t = 10 - 4t</math></p> <p><math>\Rightarrow t = 2</math></p> <p>Substituting <math>t = 2</math> in <b>both</b> expressions</p> <p>They meet at (14, 2)</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>[4]</b></p>	<p>Forming an equation for <math>t</math>. Accept vector equation for this mark. May be implied by a statement that <math>t = 2</math>.</p> <p>oe, eg showing <math>t = 2</math> satisfies both equations or a vector equation.</p> <p>Accept <math>\begin{pmatrix} 14 \\ 2 \end{pmatrix}</math></p>
<p><b>(ii)</b></p>	<p>Ashok's speed is <math>\sqrt{8^2 + 1^2} = \sqrt{65}</math></p> <p>Kumar's speed is <math>\sqrt{7^2 + (-4)^2} = \sqrt{65} \text{ km h}^{-1}</math></p> <p>They both walk at the same speed</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>[3]</b></p>	<p>CAO from correct speeds</p> <p>SC1 for finding both velocities correctly but neither speed</p>

**Q10, (Jun 2016, Q4)**

<b>(i)</b>	$\mathbf{u} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$	<b>B1</b>	
	$\mathbf{a} = \begin{pmatrix} 0 \\ -8 \end{pmatrix}$	<b>B1</b>	
		<b>[2]</b>	
<b>(ii)</b>	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$		
	$t = 2 \Rightarrow \mathbf{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 \\ -8 \end{pmatrix} \times 2$ $= \begin{pmatrix} 2 \\ -10 \end{pmatrix}$	<b>M1</b> <b>A1</b>	Or equivalent. FT for their <b>u</b> and <b>a</b>  Continue the FT for this mark
	Speed = $\sqrt{2^2 + (-10)^2} = 10.2 \text{ ms}^{-1}$ (to 3 sf)	<b>B1</b>	FT from their <b>v</b>
		<b>[3]</b>	
<b>(iii)</b>	$x = ut \Rightarrow x = 2t \Rightarrow t = \frac{x}{2}$	<b>M1</b>	This mark may also be obtained for substituting $x$ for $2t$ in the expression for $y$ .
	$y = 6t - 4t^2$	<b>B1</b>	
	$y = 6 \times \frac{x}{2} - 4 \times \left(\frac{x}{2}\right)^2 = 3x - x^2$	<b>A1</b>	
		<b>[3]</b>	