



Variable Acceleration in Two Dimensions (Sheet 2) Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(i)(a)	Integrate \mathbf{a} wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j} (+C)$	A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j} \text{ (m s}^{-1}\text{)}$	A1	1.1b
		(3)	
(i)(b)	Equate \mathbf{i} component of \mathbf{v} to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1ft	1.1b
	$t = 4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
(ii)	Differentiate \mathbf{r} wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t - 1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t = 2.5$	A1	1.1b
		(6)	
(12 marks)			

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Q2.

Question	Scheme	Marks	AOs
	Integrate \mathbf{a} w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of \mathbf{C})	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b

(6 marks)

Notes:

1st M1: for integrating \mathbf{a} w.r.t. time (powers of t increasing by 1)

1st A1: for a correct \mathbf{v} expression without \mathbf{C}

2nd A1: for a correct \mathbf{v} expression including \mathbf{C}

2nd M1: for putting $t = 4$ into their \mathbf{v} expression

3rd M1: for finding magnitude of their \mathbf{v}

3rd A1: ft for 100 m s^{-1} , follow through on an incorrect \mathbf{v}

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Q3.

Question Number	Scheme	Marks	Notes
(a)	Integrate: $\mathbf{v} = (t^3 - 2t^2)\mathbf{i} + (3t^2 - 5t)\mathbf{j} + \mathbf{C}$ $t = 3: \mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$ $\mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$ $\mathbf{v} = (t^3 - 2t^2 + 2)\mathbf{i} + (3t^2 - 5t - 2)\mathbf{j}$	M1	At least 3 powers going up. Condone errors in constants. Must be two separate component equations if not in vector form.
		A2	Could be in column vector form. Allow with no "+ C" -1 each integration error. i.e. All correct A1A1 1 error A1A0, 2 or more errors A0A0 Allow with no "+ C"
		DM1	Substitute given values to find C. Dependent on the previous M mark
		A1 (5)	Correct velocity (any equivalent form)
(b)	Parallel to $\mathbf{i} \Rightarrow 3t^2 - 5t - 2 = 0$ $(3t+1)(t-2) = 0,$ $t = 2$ $ \mathbf{v} = 8 - 8 + 2 = 2 \text{ (m s}^{-1}\text{)}$	M1	Set \mathbf{j} component of their \mathbf{v} equal to zero and solve for t . Correct answers imply method, but incorrect answers need to show method clearly.
		A1	Correct only. Ignore $-\frac{1}{3}$ if present.
		DM1	Substitute their t to find \mathbf{v} . Dependent on the previous M mark.
		A1 (4)	The answer must be a scalar – the Q asks for speed. Results from negative t must be rejected.
		[9]	
A candidate who has no "+C" can score at most M1A2M0A0 M1A0M1A0			

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Q4.

Q	Scheme	Marks
(a)	$t = \frac{5}{4}$	B1 M1 1.25
(b)	$\mathbf{r} = (2t^2 - 5t)\mathbf{i} + 3t\mathbf{j} + c$	Integrate the velocity vector NB Also correct to use suvat with $\mathbf{a} = 4\mathbf{i}$ and $\mathbf{u} = -5\mathbf{i} + 3\mathbf{j}$. Correct
(c)	$t = 0 \quad 2\mathbf{i} + 5\mathbf{j} = c$ $\mathbf{r} = (2t^2 - 5t)\mathbf{i} + 3t\mathbf{j} + (2\mathbf{i} + 5\mathbf{j})$ $(2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$ $\mathbf{r}_Q = 11\mathbf{i} + 2\mathbf{j} - 2t\mathbf{i} + ct\mathbf{j}$ $(11 - 2t)\mathbf{i} + (2 + ct)\mathbf{j}$ $\mathbf{r}_p = (2t^2 - 5t + 2)\mathbf{i} + (3t + 5)\mathbf{j}$ $\mathbf{r}_Q = \mathbf{r}_p = d\mathbf{i} + 14\mathbf{j}$ $3t + 5 = 14$ <div style="display: inline-block; vertical-align: middle; border-left: 1px solid black; padding-left: 5px; margin-left: 20px;"> $2t^2 - 3t - 9$ $(2t + 3)(t - 3) = 0$ $t = 3$ A1 ft </div> $t = 3$ $2 + ct = 14 \Rightarrow c = 4$ $d = 11 - 2 \times 3 = 5$ or $d = 2 \times 3^2 - 5 \times 3 + 2 \Rightarrow d = 5$ Alt: $2t^2 - 5t + 2 = 11 - 2t = d \Rightarrow t = \frac{11-d}{2}$ $2\left(\frac{11-d}{2}\right)^2 - 5\left(\frac{11-d}{2}\right) + 2 = d$ $d^2 - 19d + 70 = 0 = (d - 5)(d - 14)$	A1 DM1 A1 Use \mathbf{r}_0 to find C oe B1 Correct j component of \mathbf{r}_Q Do not actually require the whole thing - can answer the Q by considering only the j component. $2t^2 - 5t$ M1 Form an equation in t only A1 A1 ft Their t Their t



Q6.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i} - 8\mathbf{j}) (\text{m s}^{-1})$	A1	1.1b
		(2)	
(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = 0$) Or any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u} + \mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ (j terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate j components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1	2.1
	$T = 1.8$	A1	1.1b
		(4)	
(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the i component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	$\lambda = 2.9$ or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	
Notes: Accept column vectors throughout		(8 marks)	

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Q7.

(a)	Speed = $\sqrt{8^2 + 48^2} = \sqrt{2368} = 48.7 \text{ (ms}^{-1}\text{)}$	M1 A1	(2)
(b)	$\mathbf{a} = 2\mathbf{i} - 6t\mathbf{j}$ When $t = 4$, $\mathbf{a} = 2\mathbf{i} - 24\mathbf{j} \text{ (ms}^{-2}\text{)}$	M1 A1 A1	
(c)	$\mathbf{r} = t^2\mathbf{i} - t^3\mathbf{j} + \mathbf{C}$ $t = 1, -4\mathbf{i} + \mathbf{j} = \mathbf{i} - \mathbf{j} + \mathbf{C}, \mathbf{C} = -5\mathbf{i} + 2\mathbf{j}$ $\mathbf{r} = (t^2 - 5)\mathbf{i} + (-t^3 + 2)\mathbf{j}$ When $t = 4, \mathbf{r} = (16 - 5)\mathbf{i} + (-64 + 2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	M1 A1 DM1	(5) 10
		DM1 A1	

Q8.

Question Number	Scheme	Marks	Notes
(a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6\mathbf{i} + (4 - 2t)\mathbf{j}$ When $t = 1, \mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$ $ \mathbf{a} = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32 \text{ (m s}^{-2}\text{)}$	M1	Differentiate \mathbf{v} to obtain \mathbf{a} . Accept column vector or \mathbf{i} and \mathbf{j} components dealt with separately. Substitute $t = 1$ into their \mathbf{a} . Dependent on 1 st M1 Use of Pythagoras to find the magnitude of their \mathbf{a} . Allow with their t . Dependent on 1 st M1 Accept awrt 6.32, 6.3 or exact equivalents.
		A1	
		DM1	
		DM1	
		A1	
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j} dt$ $= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$ $t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$ When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j} \text{ (m)}$	M1	Integrate \mathbf{v} to obtain \mathbf{r} Condone C, D missing Use $t = 0, \mathbf{r} = \mathbf{i}$ to find C & D Substitute $t = 3$ with their C & D to find \mathbf{r} . Dependent on both previous Ms. cao. Must be a vector.
		A1	
		DM1	
		DM1	
		A1	
		(5) 10	



Q9.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathbf{a} = 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	M1	This mark is given for a method to differentiate the expression for \mathbf{v}
		A1	This mark is given for correctly differentiating the expression for \mathbf{v}
	$= 6\mathbf{i} - 15\mathbf{j} \text{ m s}^{-1}$	A1	This mark is given for substituting $t = 4$ to find a correct vector expression for the acceleration of P
(b)	$\mathbf{r} = (\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	M1	This mark is given for a method to integrate the expression for \mathbf{v}
		A1	This mark is given for correctly integrating the expression for \mathbf{v}
	$(-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j})$ $= 28\mathbf{i} - 44\mathbf{j} \text{ m}$	A1	This mark is given for substituting $t = 4$ to find a correct position vector of P
			(Total 6 marks)

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