Moments in 2 Dimensions (Sheet 2)

Q1.

A non-uniform rod *AB*, of mass 5kg and length 4m, rests with one end *A* on rough horizontal ground. The centre of mass of the rod is *d* metres from *A*. The rod is held in limiting equilibrium at an angle θ to the horizontal by a force **P**, which acts in a direction perpendicular to the rod at *B*, as shown in Figure 2. The line of action of **P** lies in the same vertical plane as the rod.

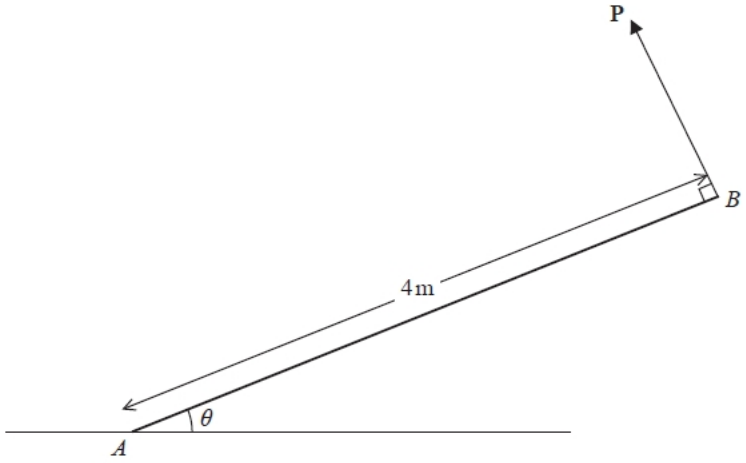


Figure 2

(a) Find, in terms of *d*, *g* and θ,

(i) the magnitude of the vertical component of the force exerted on the rod by the ground,

(ii) the magnitude of the friction force acting on the rod at *A*.

(8)

Given that $\tan\theta = \frac{5}{12}$ and that the coefficient of friction between the rod and the ground is $\frac{1}{2}$,

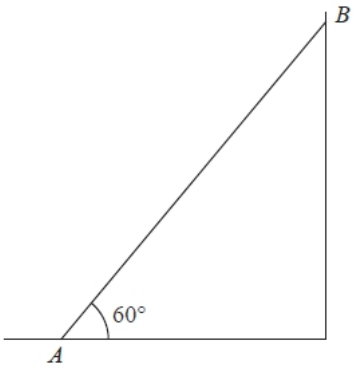
(b) find the value of *d*.

(4)

(Total for question = 12 marks)

Q2.

A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is ¼ and the coefficient of friction between the rod and the wall is ⅔. The rod is on the point of slipping at both ends.



(a) Find the magnitude of the vertical component of the force exerted on the rod by the floor.

(5)

The centre of mass of the rod is at *G*.

(b) Find the distance *AG*.

(3)

(Total 10 marks)

Q3.

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB. The rod has length $3a$ and weight W and rests on the peg at C, where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

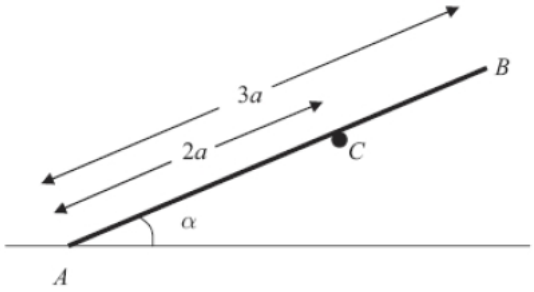


Figure 2

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3\cos^2\alpha)W$.

(6)

Given that the rod is in limiting equilibrium and that $\cos\alpha = \frac{2}{3}$

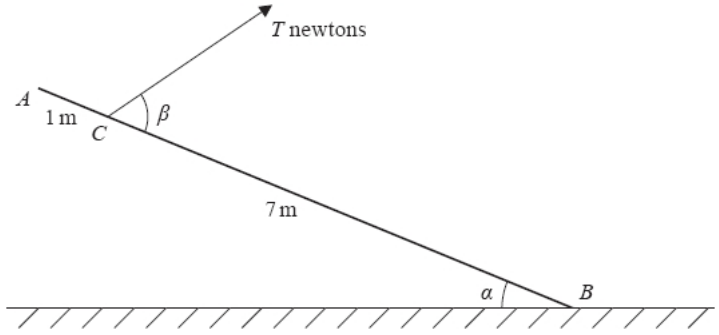
(b) find the coefficient of friction between the rod and the ground.

(5)

(Total 11 marks)

Q4.

A uniform rod AB, of mass 5 kg and length 8 m, has its end B resting on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a rope attached to the rod at C. The distance $AC = 1$ m. The rope is in the same vertical plane as the rod. The angle between the rope and the rod is β and the tension in the rope is T newtons, as shown in Figure 3. The coefficient of friction between the rod and the ground is $\frac{2}{3}$. The vertical component of the force exerted on the rod at B by the ground is R newtons.



(a) Find the value of R.

(6)

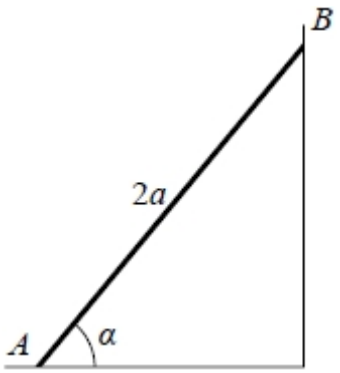
(b) Find the size of angle β.

(5)

(Total for question = 11 marks)

Q5.

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.



The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

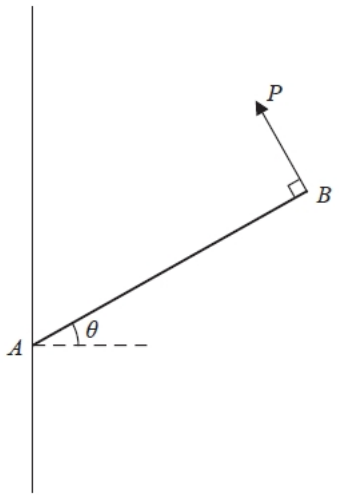
(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for question = 13 marks)

Q6.

A uniform rod AB of weight W has its end A freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle θ to the horizontal, by a force of magnitude P. The force acts perpendicular to the rod at B and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at A is Y.



(a) Show that $Y = \frac{W}{2}(2 - \cos^2\theta)$.

(6)

Given that $\theta = 45°$

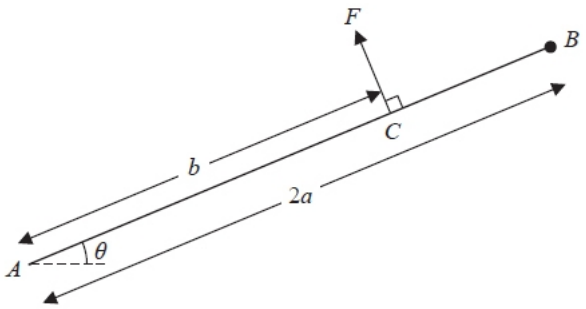
(b) find the magnitude of the force exerted on the rod by the wall at A, giving your answer in terms of W.

(6)

(Total 12 marks)

Q7.

A uniform rod AB, of mass m and length $2a$, is freely hinged to a fixed point A. A particle of mass m is attached to the rod at B. The rod is held in equilibrium at an angle θ to the horizontal by a force of magnitude F acting at the point C on the rod, where $AC = b$, as shown in Figure 3. The force at C acts at right angles to AB and in the vertical plane containing AB.



(a) Show that $F = \dfrac{3amg\cos\theta}{b}$.

(4)

(b) Find, in terms of a, b, g, m and θ,

(i) the horizontal component of the force acting on the rod at A,
(ii) the vertical component of the force acting on the rod at A.

(5)

Given that the force acting on the rod at A acts along the rod,

(c) find the value of $\dfrac{a}{b}$

(4)

(Total 13 marks)

Q8.

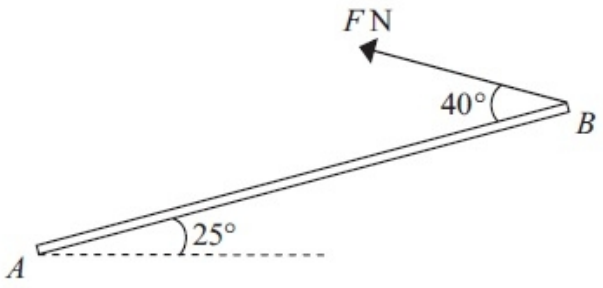


Figure 1

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25° above the horizontal by a force of magnitude F newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40° with the rod, as shown in Figure 1.

(a) Find the value of F.

(4)

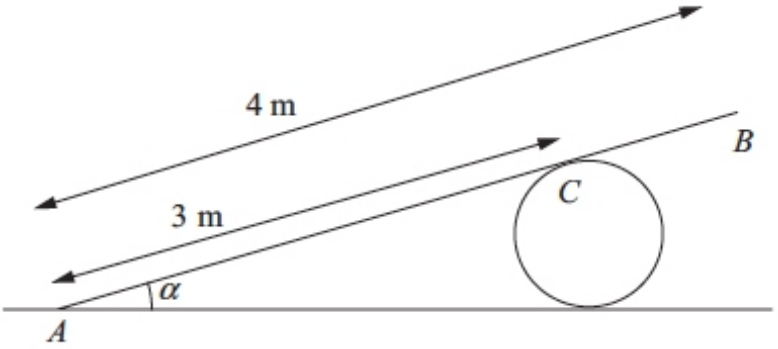
(b) Find the magnitude and direction of the vertical component of the force acting on the rod at A.

(4)

(Total 8 marks)

Q9.

A uniform plank AB, of weight 100 N and length 4 m, rests in equilibrium with the end A on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is C, where $AC = 3$ m, as shown in Figure 1. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin \alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ. Modelling the plank as a rod, find the least possible value of μ.



(10)

(Total 10 marks)

Q10.

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

(Total 10 marks)

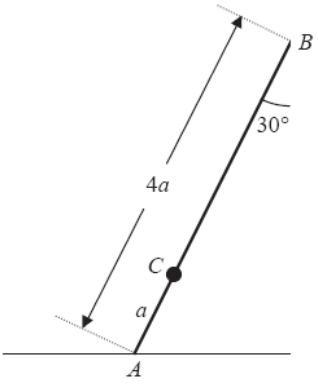
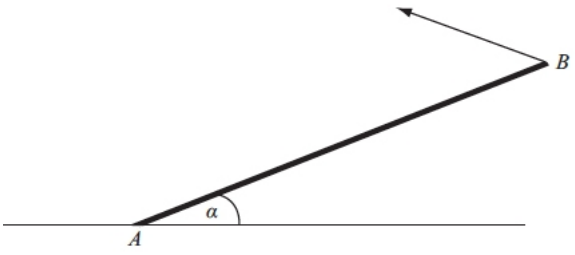


Figure 2

Q11.

Figure 2

A uniform rod AB, of mass 20 kg and length 4 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$, by a force acting at B, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at A.



(7)

(Total 7 marks)