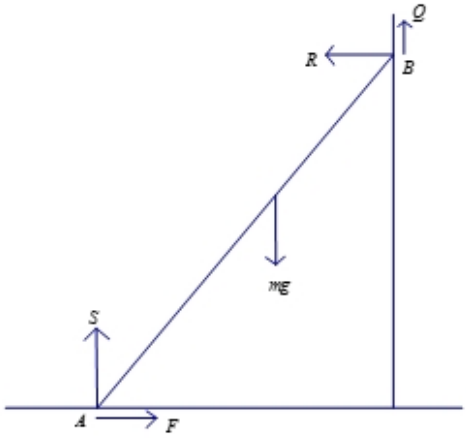
Moments in 2 Dimensions (Sheet 2) Mark Scheme

Q1.

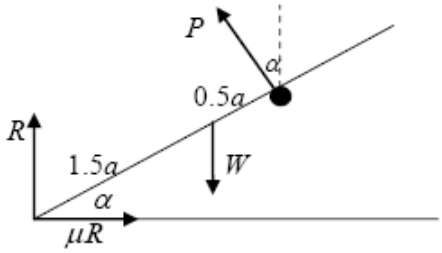
Q	Scheme	Marks	Notes
a	M(A): $d\cos\theta\times 5g = 4P$	M1	Terms must be dimensionally correct. Condone trig confusion
		A1	
	Resolving horizontally: $P\sin\theta = F$	B1	
	Resolving vertically: $P\cos\theta + R = 5g$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		A1	Correct equation
		DM1	Substitute for P to find R or F Dependent on both previous M marks
	$R = 5g - \frac{5gd\cos^2\theta}{4}$	A1	One force correct. Accept equivalent forms e.g. $R = \frac{20g - 5gd + 20g\tan^2\theta}{4\left(1+\tan^2\theta\right)}$
	$F = \frac{5gd\cos\theta\sin\theta}{4}$	A1	Both forces correct. Accept equivalent forms e.g. $F = \frac{5gd\tan\theta}{4\sec^2\theta}$
		(8)	
a alt	M(B): $5g\cos\theta\times(4-d) + F\sin\theta\times 4 = R\cos\theta\times 4$	M1	Needs all three terms. Terms must be dimensionally correct. Condone trig confusion
		A1	At most one error
	Resolve parallel to the rod: $5g\sin\theta = R\sin\theta + F\cos\theta$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		B1	At most one error
		A1	Correct equation
	$\Rightarrow R = 5g - \frac{F\cos\theta}{\sin\theta}$		
	$5g\cos\theta\times(4-d) + F\sin\theta\times 4$ $= 4\cos\theta\left(5g - \frac{F\cos\theta}{\sin\theta}\right)$	DM1	Eliminate one variable to find F or R Dependent on both previous M marks
	$4F\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)$ $= 20g\cos\theta - 20g\cos\theta + 5gd\cos\theta$		
	$F = \frac{5gd\cos\theta\sin\theta}{4}$	A1	One force correct
	$R = 5g - \frac{5gd\cos^2\theta}{4}$	A1	Both forces correct
			See next page for part (b)

b	$\mu = \dfrac{\frac{5gd\cos\theta\sin\theta}{4}}{5g - \frac{5gd\cos^2\theta}{4}}$	M1	Use of $F = \mu R$
	$\frac{1}{2}\left(5g - \frac{5gd\cos^2\theta}{4}\right) = \frac{5gd\cos\theta\sin\theta}{4}$	A1	$\left(4 - d\cos^2\theta = 2d\cos\theta\sin\theta\right)$
	$4\times169 = 120d + 144d$	M1	Use $\tan\theta = \frac{5}{12}$ and solve for d
	$d = \frac{169}{66}$	A1	(= 2.6 m or better)
		(4)	
balt	$F = 5gd \times \frac{12}{13} \times \frac{5}{13} \times \frac{1}{4}\left(= \frac{75gd}{169}\right)$	M1	Use $\tan\theta = \frac{5}{12}$
	$R = 5g - \frac{5gd}{4} \times \frac{144}{169}$	A1	Both unsimplified expressions
	$75gd = \frac{1}{2}(5\times169g - 180gd)$	M1	Use of $F = \mu R$ and solve for d
	$150gd + 180gd = 845g$, $d = \frac{169}{66}$	A1	(= 2.6 m or better)
		(4)	
balt	$R = 5g - \frac{12}{13}P$, $F = \frac{5}{13}P$	M1	Substitute trig in their equations from resolving.
	$\frac{5}{13}P = \frac{1}{2}\left(5g - \frac{12}{13}P\right)$	M1	use $F = \mu R$ and solve for d
	$\Rightarrow P = \frac{65}{22}g$	A1	
	$d = \frac{4P}{5g\cos\theta} = \frac{169}{66}$	A1	
		[12]	

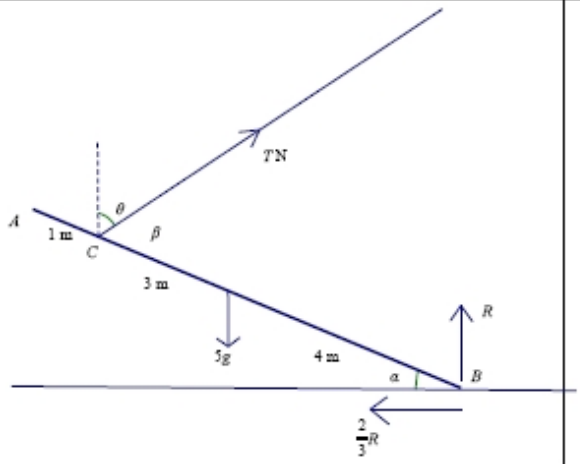
Q2.

Question Number	Scheme	Marks	Notes
			NB As the rod is not uniform, the use of moments equations is not helpful in part (a).
(a)	$R = F$	B1	Resolve horizontally
	$S + Q = mg$	B1	Resolve vertically (requires Q acting upwards)
	$Q = \frac{2}{3}R, \quad F = \frac{1}{4}S$	B1	Use both coefficients of friction
	$Q = \frac{2}{3}R = \frac{2}{3}\times\frac{1}{4}S, \qquad S + \frac{1}{6}S = mg, \ S = \frac{6}{7}mg$	M1 A1 (5)	Solve to find S in terms of m & g. (Can be scored if Q is acting downwards)
(b)	M(A) $mg \times x\cos 60 = Q \times 2l\cos 60 + R \times 2l\sin 60$	M1	Moments equation – must include all terms. Condone sign errors and sin/cos confusion
	M (B) $mg(2l - x)\cos 60 + F \times 2l\sin 60 = S \times 2l\cos 60$		
	M (c of m) $Sx\cos 60 = Fx\sin 60 + R(2l - x)\sin 60 + Q(2l - x)\cos 60$	A2	Correct unsimplified equation (for their S.) -1 each error
	$mgx\cos 60 = \frac{1}{6}\times\frac{6}{7}mg \times 2l\cos 60 + \frac{1}{4}\times\frac{6}{7}mg \times 2l\sin 60$	DM1	Form an equation in x. Depends on the preceding M
	$\frac{1}{2}x = \frac{1}{7}\times 2l \times \frac{1}{2} + \frac{3}{14}\times l\sqrt{3}$		
	$AG = x = 1.028....l \ \ x = 1.03l$	A1 (5)	$1.03l$ or better $\frac{l\left(2+3\sqrt{3}\right)}{7}$

Q3.

Question Number	Scheme	Marks
(a)		
	$R(\uparrow)\quad R+P\cos\alpha=W$	M1 A1
	$M(A)\quad P\times 2a=W\times 1.5a\cos\alpha$	M1 A1
	$\left(P=\frac{3}{4}W\cos\alpha\right)$	
	$R=W-P\cos\alpha=W-\frac{3}{4}W\cos^2\alpha$	DM1
	$=\frac{1}{4}\left(4-3\cos^2\alpha\right)W$ * cso	A1 (6)
(b)	Using $\cos\alpha=\frac{2}{3}$, $\quad R=\frac{2}{3}W$	B1
	$R(\rightarrow)\quad \mu R=P\sin\alpha$	M1 A1
	Leading to $\mu=\frac{3}{4}\sin\alpha$	
	$\left(\sin\alpha=\sqrt{\left(1-\tfrac{4}{9}\right)}=\tfrac{\sqrt{5}}{3}\right)$	
	$\mu=\frac{\sqrt{5}}{4}$ awrt 0.56	M1 A1 (5)
		(11 marks)

Q4.

Q.	Scheme	Marks	Notes
a			
	$F=\frac{2}{3}R$ seen or implied	B1	Use of $F=\mu R$. Could be on diagram. Allow in (b) if not seen before
	$\text{M}(C): 5g\times 3\cos\alpha + F\times 7\sin\alpha = 7\cos\alpha\times R$	M1	Moments about C or alternative complete method to find equation in F and R or R only. Dimensionally correct and all terms needed. Condone sin/cos confusion and sign error(s).
		A1	At most one error
		A1	Correct unsimplified equation
	$15g\cos\alpha = R\left(7\cos\alpha - \frac{14}{3}\sin\alpha\right)$		
	$15g\times\frac{4}{5} = R\left(7\times\frac{4}{5} - \frac{14}{3}\times\frac{3}{5}\right) = \frac{14}{5}R$	dM1	Substitute for F and trig and solve for R Dependent on previous M1
	$R=\frac{30}{7}g = 42\,(\text{N})$	A1	
		(6)	
	e.g. of alternative for M1A1A1:		
	$\text{M}(A): T\sin\beta + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$ **and** $\text{M}(B)$: $7T\sin\beta = 20g\cos\alpha$	(M1)	
		(A1)	At most 1 error
	$\frac{20g}{7}\cos\alpha + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$	(A1)	Correct unsimplified equation in F and R or R only

Q.	Scheme	Marks	Notes
b	Resolve ↕: $T\cos\theta + R = 5g$ $R + T\sin(\beta-\alpha) = 5g$	M1	Need all terms. Condone sin/cos confusion and sign error(s).
		A1	Correct in R or *their* R
	Resolve ↔: $T\sin\theta = F\ (=28)$ $F\left(=\frac{2}{3}R\right) = T\cos(\beta-\alpha)$	M1	Need both terms. Condone sin/cos confusion
		A1	Correct in R or *their* R
	Solve simultaneous equations for $\beta-\alpha$		
	$\tan(\beta-\alpha) = 4$, $\beta = 50.9°$ $(51°)$	A1	cso . Max 3 s.f.
		(5)	
Alt b	M(B): $7\times T\sin\beta = 5g\cos\alpha\times 4$	M1	Moments equation. Dimensionally correct. Condone sin/cos confusion and sign error(s).
	$\left(T\sin\beta = \frac{16}{7}g\right)$	A1	
	OR: resolve perpendicular to the rod: $T\sin\beta + R\cos\alpha = 5g\cos\alpha + \frac{2}{3}R\sin\alpha$	(M1) (A1)	
	Resolve parallel to rod: $T\cos\beta + 5g\sin\alpha = F\cos\alpha + R\sin\alpha$ $\left(=\frac{2}{3}R\cos\alpha + R\sin\alpha\right)$	M1	All terms needed. Condone sin/cos confusion and sign error(s).
	$\left(T\cos\beta = \frac{13}{7}g\right)$	A1	
	Solve simultaneous equations for β		
	$\tan\beta = \frac{16}{13}$, $\beta = 50.9°$ $(51°)$	A1	cso. Max 3 s.f.
		(5)	
		[11]	

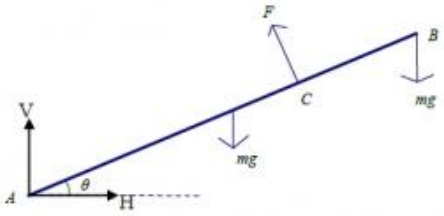
Q5.

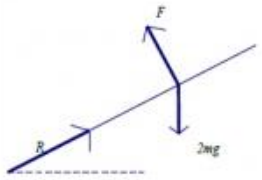
Question	Scheme	Marks	AOs
(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1	3.3
	$Wa\cos\alpha + 7W2a\cos\alpha = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Q6.

Question Number	Scheme	Marks	Notes
(a)	Resolving vertically: $Y + P\cos\theta = W$	M1	Needs all 3 terms. Condone sign errors and sin/cos confusion. Condone Wg
		A1	
	Moments about A: $Wl\cos\theta = 2lP$	M1	Terms need to be of the correct structure, but condone l implied if not seen.
		A1	
	$P = \frac{W\cos\theta}{2} \Rightarrow Y = W - \frac{W\cos^2\theta}{2} = \frac{W}{2}\left(2-\cos^2\theta\right)$ **	DM1	Substitute for P to obtain simplified Y Requires both preceding M marks
		A1 (6)	Obtain **given result** correctly.
	NB $W + Y = P\cos\theta$ with correct conclusion is possible		
	They need to find two independent equations that do not include X. If they have equations involving X they need to attempt to eliminate X before they score any marks		
(b)	$\theta = 45° \Rightarrow Y = \frac{3W}{4}$	B1	
	$X = P\sin 45$	M1	Resolving horizontally. Accept in terms of θ.
	$= \frac{W\cos 45}{2}.\sin 45\left(=\frac{W}{4}\right)$	DM1	Express X in terms of W. Accept in terms of θ. Requires preceding M mark.
		A1	Correct unsimplified but substituted.
	Resultant at $A = \frac{W}{4}\sqrt{3^2+1^2} = \frac{W\sqrt{10}}{4}$	DM1	Use of Pythagoras with X, Y in terms of W only. Dependent on the first M1
	$(0.79W)$	A1 (6)	Or equivalent ($0.79W$ or better)
	Alternative moments equations: about the centre $Pl + X\sin\theta l = y\cos\theta l$		
	About the point where the lines of action of P and X intersect $Y \times \frac{2l}{\cos\theta} = W\left(\frac{2l}{\cos\theta} - l\cos\theta\right)$		

Q7.

Question Number	Scheme	Marks	Notes
a	 Moments about A: $bF = a\cos\theta mg + 2a\cos\theta mg (= 3a\cos\theta mg)$ $F = \frac{3amg\cos\theta}{b}$ ***Answer given***	M1 A2 A1 [4]	Moments about A. Requires all three terms and terms of correct structure (force x distance). Condone consistent trig confusion -1 each error
b	$\rightarrow: \quad H = F\sin\theta = \frac{3amg\cos\theta\sin\theta}{b}$ $\uparrow: \quad 2mg = \pm V + F\cos\theta$ $\pm V = 2mg - \frac{3amg\cos\theta}{b}\times\cos\theta \left(= 2mg - \frac{3amg\cos^2\theta}{b}\right)$	M1 A1 M1 A1 A1 [5]	Resolve horizontally. Condone trig confusion RHS correct. Or equivalent. Resolve vertically. Condone sign error and trig confusion Correct equation RHS correct. Or equivalent

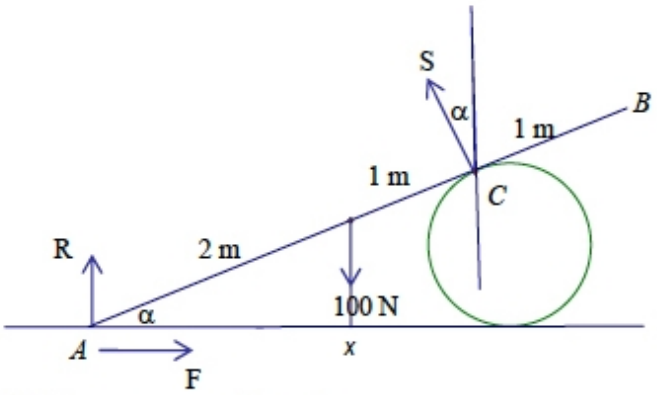
Question Number	Scheme	Marks	Notes
c	$\frac{2mg - \frac{3amg\cos^2\theta}{b}}{\frac{3amg\cos\theta\sin\theta}{b}} = \tan\theta$ $\frac{2b - 3a\cos^2\theta}{3a\cos\theta\sin\theta} = \frac{\sin\theta}{\cos\theta}$ $\Rightarrow 2b - 3a\cos^2\theta = 3a\sin^2\theta \Rightarrow 2b = 3a, \quad \frac{a}{b} = \frac{2}{3}$	M1 A1 DM1 A1 [4]	Use of tan, either way up. *V, H, F* substituted. Correct for their components in θ only Simplify to obtain the ratio of a and b, or equivalent
c alt 2	The centre of mass of the combined rod + particle is $\frac{3}{2}a$ from *A*  3 forces in equilibrium must be concurrent $\Rightarrow b = \frac{3}{2}a$ $\Rightarrow \frac{a}{b} = \frac{2}{3}$	M1A1 M1 A1 [4]	Not on the spec, but you might see it.
alt c 3	*R* acts along the rod, so resolve forces perpendicular to the rod. $F = mg\cos\theta + mg\cos\theta$ $2mg\cos\theta = \frac{3amg\cos\theta}{b}$ $\Rightarrow \frac{a}{b} = \frac{2}{3}$	M1 A1 DM1 A1 [4]	Resolve and substitute for *F* Eliminate θ

Question Number	Scheme	Marks	Notes
alt c 4	R acts along the rod. Take moments about C $mg\cos\theta\ \ 2a-b\ \ =mg\cos\theta\ \ b-a$ $2a-b=b-a,\quad \Rightarrow \frac{a}{b}=\frac{2}{3}$	M1 A1 DM1A1 [4]	Moments about B gives $2a-b\ \ F=amg\cos\theta$ and substitute for F
c alt 5	Resultant parallel to the rod $\Rightarrow R=2mg\sin\theta$ And $V^2+H^2=R^2$ $2mg\sin\theta\ ^2=\left(\frac{3amg\cos\theta\sin\theta}{b}\right)^2+\left(2mg-\frac{3amg\cos^2\theta}{b}\right)^2$ Eliminate θ $\Rightarrow \frac{a}{b}=\frac{2}{3}$	M1 A1 DM1 A1 [4]	Substitute for V, H and R in terms of θ

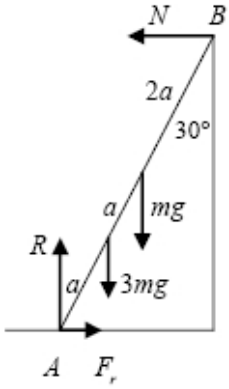
Q8.

Question Number	Scheme	Marks	Notes
(a)	$M(A), F.4\sin 40^\circ = 5g.2\cos 25^\circ$	M1	A complete method to find F, e.g. take moments about A. Condone sin/cos confusion. Requires correct ratio of lengths.
		A1	Correct terms with at most one slip
		A1	All correct
	$F=35$	A1	35 or 34.5 (>3sf not acceptable due to use of 9.8, but only penalise once in a question)
		(4)	
(b)	$F\cos 75^\circ \pm Y = 5g$	M1	Resolve vertically. Need all three terms but condone sign errors. Must be attempting to work with their 75° or 15°.
		A1	Correct equation (their F)
	$Y=40$;	A1	40 or 40.1
	UP	A1	Apply ISW if the candidate goes on to find R. cso (the Q does specifically ask for the direction, so this must be clearly stated)
		(4)	
		8	
(b)	OR1: $4m\cos 25\times Y$ $=5g\times 2m\cos 25+F\cos 15\times 4m\sin 25$ etc.	M1 A1	Taking moments about the point vertically below B and on the same horizontal level as A. (Their F)
	OR2: $R\cos\alpha = F\cos 40+5g\cos 65$ $R\sin\alpha + F\sin 40 = 5g\cos 25$		Resolve parallel & perpendicular to the rod
	$R=52.1,\ \alpha=25.3^\circ$		Solve for R, α
	$Y=R\sin(25+\alpha)$ Etc.	M1A1	Need a complete strategy to find Y for M1.

Q9.

Question Number	Scheme	Marks
	 Taking moments about A: $3S = 100 \times 2 \times \cos\alpha$	M1 A1
	Resolving vertically: $R + S\cos\alpha = 100$	M1 A1
	Resolving horizontally: $S\sin\alpha = F$	M1 A1
	(Most alternative methods need 3 independent equations, each one worth M1A1. Can be done in 2 e.g. if they resolve horizontally and take moments about X then $R \times 2 \times \cos\alpha = S \times (3 - 2 \times \cos^2\alpha)$ scores M2A2)	
	Substitute trig values to obtain correct values for F and R (exact or decimal equivalent). $\left(S = \frac{200\sqrt{8}}{9}\right)$, $R = 100 - \frac{1600}{27} = \frac{1100}{27} \approx 40.74$, $F = \frac{200\sqrt{8}}{27} \approx 20.95...$	DM1 A1
	$F \le \mu R$, $200\sqrt{8} \le \mu \times 1100$, $\mu \ge \frac{200\sqrt{8}}{1100} = \frac{2\sqrt{8}}{11}$. Least possible μ is 0.514 (3sf), or exact.	M1 A1 **[10]**

Q10.

Question Number	Scheme	Marks
	(a) 	
	$\mathcal{M}(A)$ $N \times 4a\cos 30° = 3mg \times a\sin 30° + mg \times 2a\sin 30°$	M1 A2(1,0)
	$N = \frac{5}{4}mg\tan 30°$ $\left(= \frac{5}{4\sqrt{3}}mg = 7.07\ldots \text{m}\right)$	DM1 A1
	$\rightarrow \quad F_r = N \quad , \quad \uparrow \quad R = 4mg$	B1, B1
	Using $F_r = \mu R$	B1
	$\frac{5}{4\sqrt{3}}mg = \mu R$ for their R	M1
	$\mu = \frac{5}{16\sqrt{3}}$ awrt 0.18	A1 **(10)**
		[10]
	Alternative method:	
	M(B): $mg \times 2a\sin 30 + 3mg \times 3a\sin 30 + F \times 4a\cos 30 = R \times 4a\sin 30$	
	$11mga\sin 30 + F \times 4a\cos 30 = R \times 4a\sin 30$	M1A3(2,1,0)
	$\frac{11mg}{2} + F\frac{4\sqrt{3}}{2} = 2R$	DM1A1
	$\uparrow \quad R = 4mg$,	B1
	Using $F_r = \mu R$	B1
	$8\mu\sqrt{3} = \frac{5}{2}, \quad \mu = \frac{5}{16\sqrt{3}}$	M1 A1

Q11.

	$m(B): R \times 4\cos\alpha = F \times 4\sin\alpha + 20g \times 2\cos\alpha$	M1 A2
	Use of $F = \frac{1}{2}R$	M1
	Use of correct trig ratios	B1
	R = 160N or 157N	DM1 A1
		[7]