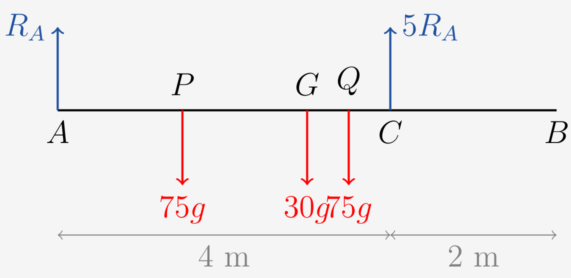
Question 1

Worked Solution

Force diagram:



Part (a): Find x.

The plank has $CB = 2$ m, $AB = 6$ m, so $AC = 4$ m.
G is the midpoint of AB, so $AG = 3$ m.
Let $R_A = R$ and $R_C = 5R$.

Resolve vertically (↑):

$$R + 5R = 75g + 30g + 75g \implies 6R = 180g \implies R = 30g$$

Moments about A:

$$75g \cdot x + 30g \cdot 3 + 75g \cdot 2x = 5R \cdot 4$$

$$75gx + 90g + 150gx = 600g$$

$$225gx = 510g$$

$$x = \frac{510}{225} = \frac{34}{15} \approx 2.3$$

$$x = \frac{34}{15} \approx 2.3$$

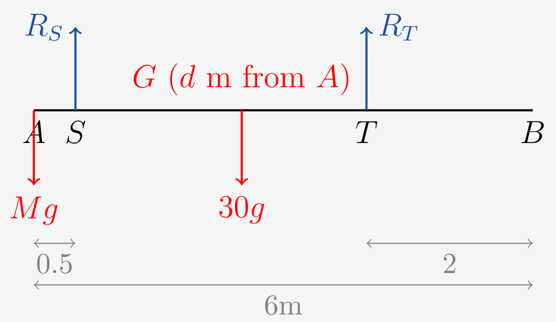
Part (b): Two assumptions from modelling as a uniform rod.

- The mass of the plank acts at its midpoint (centre of mass at the middle of the plank).
- The plank does not bend / remains straight / is rigid.

Question 2

Worked Solution

Force diagram:



Setup: $AS = 0.5$ m, $TB = 2$ m, so $AT = 4$ m; plank length $= 6$ m.

Block at A, tilting about S (so $R_T = 0$):

Moments about S:

$$Mg \times 0.5 = 30g(d - 0.5)$$

$$0.5M = 30(d - 0.5) \quad \cdots (1)$$

Block at B, tilting about T (so $R_S = 0$):

Moments about T:

$$Mg \times 2 = 30g(4 - d)$$

$$2M = 30(4 - d) \quad \cdots (2)$$

(i) Find d: Divide (2) by (1):

$$\frac{2M}{0.5M} = \frac{30(4-d)}{30(d-0.5)} \implies 4 = \frac{4-d}{d-0.5}$$

$$4(d - 0.5) = 4 - d \implies 4d - 2 = 4 - d \implies 5d = 6$$

$d = 1.2$

(ii) Find M: Substitute $d = 1.2$ into (1):

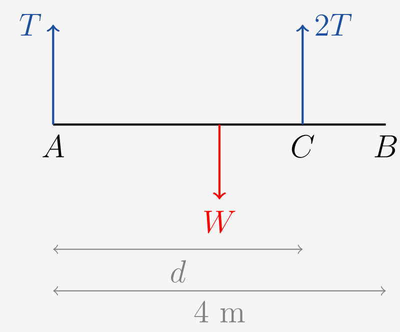
$$0.5M = 30(1.2 - 0.5) = 30 \times 0.7 = 21 \implies M = 42$$

$M = 42$

Question 3

Worked Solution

Force diagram:



Part (a): Find d.

Uniform beam: weight W acts at midpoint (2 m from A). Let tension at $A = T$, at $C = 2T$.

Resolve vertically:

$$T + 2T = W \implies 3T = W \implies T = \frac{W}{3}$$

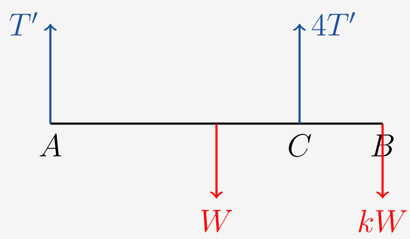
Moments about A:

$$2T \times d = W \times 2$$

$$\frac{2W}{3} \cdot d = 2W \implies d = 3$$

$d = 3$ m

Part (b): Load kW added at B; tension at C now $= 4T'$ where $T' =$ tension at A.



Resolve vertically:

$$T' + 4T' = W + kW \implies 5T' = W(1 + k)$$

Moments about A:

$$2W + 4kW = 3 \times 4T' = 12T'$$

Substitute $T' = \dfrac{W(1+k)}{5}$:

$$2W + 4kW = \frac{12W(1+k)}{5}$$

$$5(2+4k) = 12(1+k)$$

$$10 + 20k = 12 + 12k$$

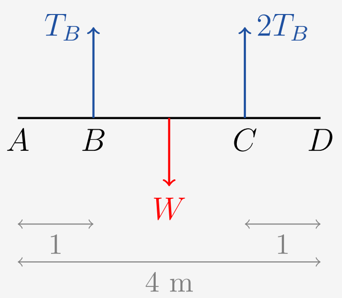
$$8k = 2$$

$k = \dfrac{1}{4}$

Question 4

Worked Solution

Force diagram:



Part (a): Find distance of centre of mass from A.

$AB = 1$ m, $CD = 1$ m, so $BC = 2$ m, $AC = 3$ m. Let $T_B = T$, $T_C = 2T$, centre of mass at distance $\bar{x}$ from A.

Resolve vertically:

$$T + 2T = W \implies T = \tfrac{W}{3}$$

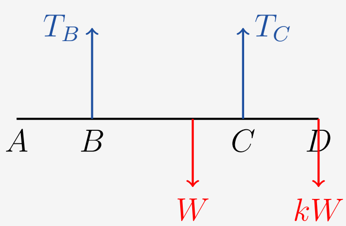
Moments about B:

$$2T \times 2 = W(\bar{x} - 1)$$

$$\frac{4W}{3} = W(\bar{x} - 1) \implies \bar{x} - 1 = \frac{4}{3} \implies \bar{x} = \frac{7}{3}$$

Distance of centre of mass from $A = \dfrac{7}{3}$ m

Part (b): Load kW at D; expression for T_B.



Moments about C:

$$T_B \times 2 + kW \times 1 = W \times \left(\frac{7}{3} - 3\right) \times (-1)$$

More carefully, moments about C (taking clockwise positive, C is at 3 m from A):

$$W\left(3 - \tfrac{7}{3}\right) = T_B \times 2 + kW \times 1$$

Wait — G is at $\frac{7}{3}$ m from A, so $\frac{7}{3} < 3$, meaning G is to the left of C.

Moments about C (anticlockwise = clockwise):

$$T_B \times 2 = W\left(3 - \tfrac{7}{3}\right) - kW \times 1$$

$$2T_B = W \cdot \tfrac{2}{3} - kW = W\left(\tfrac{2}{3} - k\right) = W \cdot \tfrac{2-3k}{3}$$

$$T_B = W\frac{(2-3k)}{6}$$

Part (c): Values of k for both ropes taut.

Both $T_B \geq 0$ and $T_C \geq 0$ required.

$T_B \geq 0 \implies 2 - 3k \geq 0 \implies k \leq \dfrac{2}{3}$

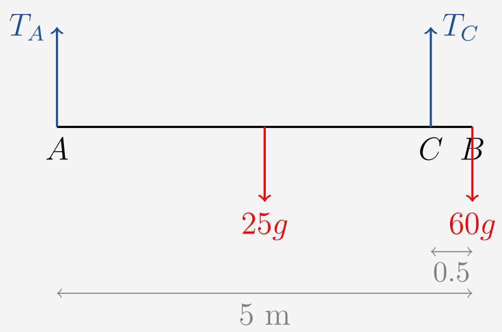
Also $k > 0$ for a positive load, and both supports remain in contact, so:

$$0 < k \leq \frac{2}{3}$$

Question 5

Worked Solution

Force diagram:



Part (a): Find T_A and T_C.

$AB = 5$ m, $CB = 0.5$ m, so $AC = 4.5$ m. Uniform beam: weight at 2.5 m from A.

Moments about C:

$$T_A \times 4.5 + 60g \times 0.5 = 25g \times 2$$

Wait — moments about C: (C is at 4.5 m from A)

$$T_A \times 4.5 = 25g \times (4.5 - 2.5) + 60g \times 0.5$$

Hmm — let me take moments about A:

$$T_C \times 4.5 = 25g \times 2.5 + 60g \times 5$$

$$T_C = \frac{62.5g + 300g}{4.5} = \frac{362.5g}{4.5} = \frac{725g}{9} \approx 790 \text{ N}$$

Resolve vertically:

$$T_A + T_C = 25g + 60g = 85g$$

$$T_A = 85g - \frac{725g}{9} = \frac{765g - 725g}{9} = \frac{40g}{9} \approx 43.6 \text{ N}$$

$$T_A = \frac{40g}{9} \approx 43.6 \text{ N}, \quad T_C = \frac{725g}{9} \approx 790 \text{ N}$$

Part (b): Particle Q of mass M kg at B.

For equilibrium both ropes must be taut: $T_A \geq 0$.

Moments about C (for $T_A \geq 0$, limiting case $T_A = 0$):

$$0 = 25g \times (4.5 - 2.5) - Mg \times 0.5$$

$$25g \times 2 = Mg \times 0.5 \implies M = 100$$

(i) Greatest possible value of M:

$M_{\text{max}} = 100$ kg

(ii) Greatest possible tension in rope at C (when $M = 100$, $T_A = 0$):

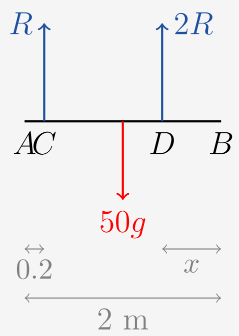
$$T_C = 25g + 100g = 125g \approx 1230 \text{ N}$$

Greatest $T_C = 125g \approx 1230$ N

Question 6

Worked Solution

Force diagram (part a):



Part (a): Find x.

Uniform rod, $AC = 0.2$ m, $DB = x$ m, so $AD = 2 - x$. Weight acts at midpoint (1 m from A). Let $R_C = R$, $R_D = 2R$.

Resolve vertically:

$$R + 2R = 50g \implies R = \frac{50g}{3}$$

Moments about C:

$$50g \times (1 - 0.2) = 2R \times (2 - x - 0.2)$$

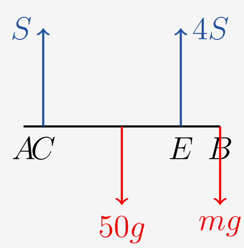
$$50g \times 0.8 = \frac{100g}{3}(1.8 - x)$$

$$40 = \frac{100(1.8 - x)}{3} \implies 120 = 100(1.8 - x)$$

$$1.8 - x = 1.2 \implies x = 0.6$$

$x = 0.6$ m

Part (b): Support at E where $EB = 0.4$ m, so $AE = 1.6$ m. Particle mass m at B. $R_E = 4R_C$.



Resolve vertically:

$$S + 4S = (50 + m)g \implies 5S = (50 + m)g$$

Moments about B:

$$50g \times 1 = 4S \times 0.4 + S \times 1.8$$

$$50g = 1.6S + 1.8S = 3.4S \implies S = \frac{50g}{3.4}$$

From vertical resolution:

$$(50 + m) = \frac{5 \times 50}{3.4} = \frac{250}{3.4} = \frac{1250}{17}$$

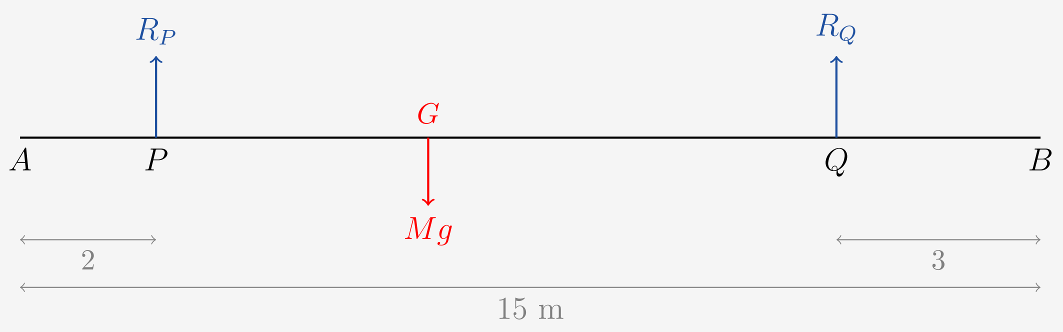
$$m = \frac{1250}{17} - 50 = \frac{1250 - 850}{17} = \frac{400}{17} \approx 23.5$$

$$m = \frac{400}{17} \approx 23.5 \text{ kg}$$

Question 7

Worked Solution

Force diagram:



Part (a): Find mass M and distance x of G from A.

Child at A, tilting about P ($R_Q = 0$):

$$\text{M}(P): \quad 50g \times 2 = Mg(x-2)$$

$$100 = M(x-2) \quad \cdots (1)$$

Child at B, tilting about Q ($R_P = 0$):

$$\text{M}(Q): \quad 50g \times 3 = Mg(12-x)$$

$$150 = M(12-x) \quad \cdots (2)$$

Divide (2) by (1):

$$\frac{3}{2} = \frac{12-x}{x-2} \implies 3(x-2) = 2(12-x) \implies 3x-6 = 24-2x \implies 5x = 30$$

$$x = 6 \text{ m from } A$$

Substitute into (1): $100 = M(6-2) = 4M \implies M = 25$ kg.

Mass of beam = 25 kg; centre of mass is 6 m from A

Part (b): Child at X, equal reactions.

Let $R_P = R_Q = R$. Resolve vertically: $2R = 25g + 50g = 75g \implies R = 37.5g$.

Moments about A:

$$2R \times 2 + 12R = 25g \times 6 + 50g \times AX$$

Hmm — use M(A) with R_P at P (2 m) and R_Q at Q (12 m):

$$R \times 2 + R \times 12 = 25g \times 6 + 50g \times AX$$

$$14R = 150g + 50g \cdot AX$$

$$14 \times 37.5g = 150g + 50g \cdot AX$$

$$525g = 150g + 50g \cdot AX$$

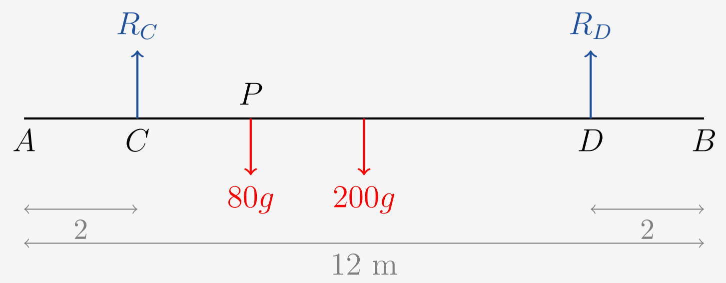
$$50 \cdot AX = 375 \implies AX = 7.5 \text{ m}$$

$AX = 7.5$ m

Question 8

Worked Solution

Force diagram (part a):



Part (a): Find R_C.

Uniform girder: G at 6 m from A. Man at P, $AP = 4$ m.

Moments about D:

$$8R_C = 80g \times 6 + 200g \times 4$$

$$8R_C = 480g + 800g = 1280g$$

Wait — distances from D: D is at 10 m from A.
G is at 6 m from $A \to D$ is $10 - 6 = 4$ m to the right of G.
P is at 4 m from $A \to D$ is $10 - 4 = 6$ m to the right of P.
C is at 2 m from $A \to D$ is $10 - 2 = 8$ m to the right of C.

M(D):

$$8R_C = 80g \times 6 + 200g \times 4$$

$$8R_C = 480g + 800g = 1280g \implies R_C = 160g \approx 1570 \text{ N}$$

$R_C = 160g \approx 1570$ N

Part (b): Support moved to X where $XB = x$ m; reactions equal.

Total weight $= 80g + 200g = 280g$; each reaction $= 140g$.

Reaction at $X = 140g \approx 1370$ N

Part (c): Find x.

C is at 2 m from A; X is at $(12 - x)$ m from A.

M(B):

$$S \cdot x + S \cdot 10 = 80g \times 8 + 200g \times 6$$

where $S = 140g$ and the C support is at $B - 10 = 2$ m from A, so 10 m from B; X is x m from B.

$$140g \cdot x + 140g \cdot 10 = 640g + 1200g = 1840g$$

$$140x + 1400 = 1840$$

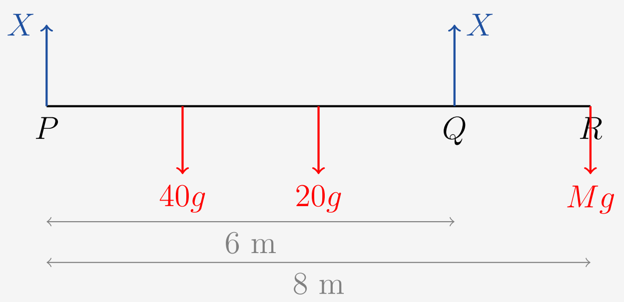
$$140x = 440 \implies x = \frac{440}{140} = \frac{22}{7} \approx 3.14 \text{ m}$$

$x = \dfrac{22}{7} \approx 3.14$ m

Question 9

Worked Solution

Force diagram:



$PQ = 6$ m, $PR = 8$ m; uniform rod so G at 4 m from P. Child at 2 m from P. Block at R. Forces at P and Q equal $= X$.

Part (a)(i): Magnitude of force at P.

Resolve vertically:

$$2X = 40g + 20g + Mg \quad \cdots (1)$$

Moments about R:

$$X \times 8 + X \times 2 = 40g \times 6 + 20g \times 4$$

$$10X = 240g + 80g = 320g$$

$$X = 32g \approx 314 \text{ N}$$

Force at $P = 32g \approx 314$ N

Part (a)(ii): Find M.

From (1): $2 \times 32g = (60 + M)g \implies 64 = 60 + M \implies M = 4$

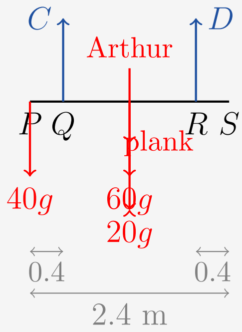
$M = 4$ kg

Part (b): The child and block are modelled as particles: their weights act at a single point (the position given), rather than being distributed.

Question 10

Worked Solution

Force diagram (part a):



Part (a): $PQ = QS =$ symmetric; legs at Q (0.4 m from P) and R (0.4 m from S, i.e. 2.0 m from P). Arthur at midpoint (1.2 m from P), Beatrice at P.

Resolve vertically:

$$C + D = 40g + 60g + 20g = 120g$$

Moments about Q:

$$80g \times 0.4 - 20g \times 0.8 + 60g \times 0.8 = D \times 1.6$$

Distances from Q: Beatrice (P) is 0.4 m left; plank G is 0.8 m right; Arthur is 0.8 m right; R is 1.6 m right.

$$D \times 1.6 = 60g \times 0.8 + 20g \times 0.8 - 40g \times 0.4$$

$$1.6D = 48g + 16g - 16g = 48g \implies D = 30g \approx 294 \text{ N}$$

$$C = 120g - 30g = 90g \approx 882 \text{ N}$$

Reaction at Q: $C = 90g \approx 882$ N; Reaction at R: $D = 30g \approx 294$ N

Part (b): Arthur moves to X. $C = 2D$. Beatrice still at P.

Resolve vertically:

$$2D + D = 120g \implies D = 40g, \quad C = 80g$$

Moments about Q:

$$C \times 0 + D \times 1.6 = 40g \times 0.4 + 60g \times QX + 20g \times 0.8$$

Wait: taking moments about Q (all distances from Q):

$$D \times 1.6 = -40g \times 0.4 + 20g \times 0.8 + 60g \times QX$$

Beatrice at P is 0.4 m to the left of Q (anticlockwise moment).
Plank G is 0.8 m to the right of Q (clockwise).
Arthur at X is QX to the right of Q.

$$40g \times 1.6 = 40g \times 0.4 - 20g \times 0.8 + 60g \times QX$$

Moments about Q (clockwise positive):

$$D \times 1.6 + 40g \times 0.4 = 20g \times 0.8 + 60g \times QX$$

Wait — Beatrice is at P which is 0.4 m to the LEFT of Q. Her weight creates anticlockwise moment about Q.

Moments about Q (clockwise positive):

$$D \times 1.6 = 20g \times 0.8 + 60g \times QX - 40g \times 0.4$$

$$40g \times 1.6 = 16g + 60g \cdot QX - 16g$$

$$64g = 60g \cdot QX$$

$$QX = \frac{64}{60} = \frac{16}{15} \approx 1.07 \text{ m}$$

$$QX = \frac{16}{15} \approx 1.07 \text{ m}$$

End of Worked Solutions