# Question 1

## Worked Solution

A particle is in equilibrium under forces $\begin{pmatrix} x \\ -7 \\ z \end{pmatrix}$, $\begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$ (in newtons).

**Part (i): Find x, y and z**

Equilibrium: sum of forces = $\mathbf{0}$.

$x$-component: $x + 4 + 5 = 0 \implies x = -9$

$y$-component: $-7 + y + 4 = 0 \implies y = 3$

$z$-component: $z - 5 - 7 = 0 \implies z = 12$

$x = -9, \quad y = 3, \quad z = 12$

**Part (ii): Calculate the magnitude of $\begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix}$**

$$\left| \begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} \right| = \sqrt{5^2 + 4^2 + (-7)^2} = \sqrt{25 + 16 + 49} = \sqrt{90} \approx 9.49 \text{ N}$$

Magnitude $= \sqrt{90} \approx 9.49$ N

## Question 2

### Worked Solution

$\mathbf{F} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ N, $\mathbf{G} = \begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$ N.

**Part (i): Resultant of F and G, and its magnitude**

$$\mathbf{F} + \mathbf{G} = \begin{pmatrix} 4-6 \\ 1+2 \\ 2+4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \text{ N}$$

$$\left| \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \right| = \sqrt{4+9+36} = \sqrt{49} = 7 \text{ N}$$

Resultant $= \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$ N, magnitude $= 7$ N

**Part (ii): Find H such that F, 2G and H are in equilibrium**

$\mathbf{F} + 2\mathbf{G} + \mathbf{H} = \mathbf{0}$

$$2\mathbf{G} = \begin{pmatrix} -12 \\ 4 \\ 8 \end{pmatrix}$$

$$\mathbf{H} = -\mathbf{F} - 2\mathbf{G} = -\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -12 \\ 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -4+12 \\ -1-4 \\ -2-8 \end{pmatrix} = \begin{pmatrix} 8 \\ -5 \\ -10 \end{pmatrix} \text{ N}$$

$\mathbf{H} = \begin{pmatrix} 8 \\ -5 \\ -10 \end{pmatrix}$ N

## Question 3

### Worked Solution

Three forces on a body of mass 4 kg: $\begin{pmatrix}-1\\14\\-8\end{pmatrix}$ N, $\begin{pmatrix}3\\-9\\10\end{pmatrix}$ N, $\mathbf{F}$ N.

Resulting acceleration $\begin{pmatrix}-1\\2\\4\end{pmatrix}$ $\text{m s}^{-2}$.

**Part (i): Calculate F**

By Newton's second law: $\begin{pmatrix}-1\\14\\-8\end{pmatrix} + \begin{pmatrix}3\\-9\\10\end{pmatrix} + \mathbf{F} = 4\begin{pmatrix}-1\\2\\4\end{pmatrix}$

$$\begin{pmatrix}2\\5\\2\end{pmatrix} + \mathbf{F} = \begin{pmatrix}-4\\8\\16\end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix}-4-2\\8-5\\16-2\end{pmatrix} = \begin{pmatrix}-6\\3\\14\end{pmatrix} \text{ N}$$

$$\mathbf{F} = \begin{pmatrix}-6\\3\\14\end{pmatrix} \text{ N}$$

**Part (ii): Velocity and speed 3 seconds later**

Initial velocity $\mathbf{u} = \begin{pmatrix}-3\\3\\6\end{pmatrix}$ $\text{m s}^{-1}$, $\mathbf{a} = \begin{pmatrix}-1\\2\\4\end{pmatrix}$ $\text{m s}^{-2}$.

$$\mathbf{v} = \begin{pmatrix}-3\\3\\6\end{pmatrix} + 3\begin{pmatrix}-1\\2\\4\end{pmatrix} = \begin{pmatrix}-3-3\\3+6\\6+12\end{pmatrix} = \begin{pmatrix}-6\\9\\18\end{pmatrix} \text{ m s}^{-1}$$

$$|\mathbf{v}| = \sqrt{36+81+324} = \sqrt{441} = 21 \text{ m s}^{-1}$$

$$\mathbf{v} = \begin{pmatrix}-6\\9\\18\end{pmatrix} \text{ m s}^{-1}, \text{ speed} = 21 \text{ m s}^{-1}$$

## Question 4

**Worked Solution**

$\mathbf{F} = \begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$ N, $\mathbf{G} = \begin{pmatrix} -6 \\ y \\ z \end{pmatrix}$ N, $\mathbf{H} = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$ N.

**Part (i): F and G act in parallel lines — find y and z**

**F** and **G** are parallel: $\mathbf{G} = k\mathbf{F}$ for some scalar $k$.

From $x$-components: $-6 = k(-2) \implies k = 3$.

$$y = 3 \times 3 = 9, \quad z = 3 \times (-4) = -12$$

$y = 9, \quad z = -12$

**Part (ii): Acceleration of object of mass 5 kg under F and H only**

$$\mathbf{F} + \mathbf{H} = \begin{pmatrix} -2+3 \\ 3-5 \\ -4-1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} \text{ N}$$

$$\mathbf{a} = \frac{\mathbf{F}+\mathbf{H}}{5} = \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix} \text{ m s}^{-2}$$

$$|\mathbf{a}| = \sqrt{0.04 + 0.16 + 1} = \sqrt{1.2} \approx 1.10 \text{ m s}^{-2}$$

$\mathbf{a} = \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ m s$^{-2}$, magnitude $\approx 1.10$ m s$^{-2}$

# Question 5

## Worked Solution

Unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ (east), $\begin{pmatrix}0\\1\\0\end{pmatrix}$ (north), $\begin{pmatrix}0\\0\\1\end{pmatrix}$ (vertically up).

Alesha (mass 100 kg), sky-dive. Initial position $\begin{pmatrix}-75\\90\\750\end{pmatrix}$ m, initial velocity $\begin{pmatrix}-5\\0\\-10\end{pmatrix}$ m s$^{-1}$.

Forces: $\begin{pmatrix}0\\0\\-980\end{pmatrix}$ N (weight), $\begin{pmatrix}0\\0\\880\end{pmatrix}$ N (air resistance), $\begin{pmatrix}50\\-20\\0\end{pmatrix}$ N (power unit).

**Part (ii): Suggest how the forces arise**

$\begin{pmatrix}0\\0\\-980\end{pmatrix}$ N: weight (gravity pulling downwards). $\begin{pmatrix}0\\0\\880\end{pmatrix}$ N: air resistance opposing vertical descent (arms spread to slow fall). $\begin{pmatrix}50\\-20\\0\end{pmatrix}$ N: thrust from the power unit strapped to her back, providing horizontal acceleration.

**Part (iii): Find acceleration and show magnitude is 1.14 m s$^{-2}$**

$$\text{Total force} = \begin{pmatrix}0+0+50\\0+0-20\\-980+880+0\end{pmatrix} = \begin{pmatrix}50\\-20\\-100\end{pmatrix} \text{ N}$$

$$\mathbf{a} = \frac{1}{100}\begin{pmatrix}50\\-20\\-100\end{pmatrix} = \begin{pmatrix}0.5\\-0.2\\-1\end{pmatrix} \text{ m s}^{-2}$$

$$|\mathbf{a}| = \sqrt{0.25+0.04+1} = \sqrt{1.29} \approx 1.136 \approx 1.14 \text{ m s}^{-2} \checkmark$$

$\mathbf{a} = \begin{pmatrix}0.5\\-0.2\\-1\end{pmatrix}$ m s$^{-2}$, $|\mathbf{a}| \approx 1.14$ m s$^{-2}$

**Part (iv): Velocity and position at time t; verify position at $t = 30$**

$$\mathbf{v} = \begin{pmatrix}-5\\0\\-10\end{pmatrix} + t\begin{pmatrix}0.5\\-0.2\\-1\end{pmatrix} = \begin{pmatrix}-5+0.5t\\-0.2t\\-10-t\end{pmatrix} \text{ m s}^{-1}$$

$$\mathbf{r} = \begin{pmatrix} -75 \\ 90 \\ 750 \end{pmatrix} + \begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0.5 \\ -0.2 \\ -1 \end{pmatrix} t^2 = \begin{pmatrix} -75 - 5t + 0.25t^2 \\ 90 - 0.1t^2 \\ 750 - 10t - 0.5t^2 \end{pmatrix} \text{ m}$$

At $t = 30$:

$$x : -75 - 150 + 0.25(900) = -75 - 150 + 225 = 0 \checkmark$$

$$y : 90 - 0.1(900) = 90 - 90 = 0 \checkmark$$

$$z : 750 - 300 - 0.5(900) = 750 - 300 - 450 = 0 \checkmark$$

$\mathbf{v} = \begin{pmatrix} -5 + 0.5t \\ -0.2t \\ -10 - t \end{pmatrix}$ m s$^{-1}$, $\mathbf{r} = \begin{pmatrix} -75 - 5t + 0.25t^2 \\ 90 - 0.1t^2 \\ 750 - 10t - 0.5t^2 \end{pmatrix}$ m

At $t = 30$: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, confirming she is at the origin. ✓

**Part (v): Why the model is unrealistic**

At $t = 30$, the vertical component of velocity is $-10 - 30 = -40$ m s$^{-1}$. This is far too fast for a safe landing (it corresponds to a speed of about 144 km/h), so the model is unrealistic.

## Question 6

### Worked Solution

**Take $g = 10$ m s$^{-2}$.**

Unit vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ (east), $\begin{pmatrix}0\\1\\0\end{pmatrix}$ (north), $\begin{pmatrix}0\\0\\1\end{pmatrix}$ (up).

Forces: $\mathbf{p} = \begin{pmatrix}-1\\-1\\5\end{pmatrix}$ N, $\mathbf{q} = \begin{pmatrix}-1\\-4\\2\end{pmatrix}$ N, $\mathbf{r} = \begin{pmatrix}2\\5\\0\end{pmatrix}$ N.

**Part (i): Which has greatest magnitude?**

$$|\mathbf{p}| = \sqrt{1+1+25} = \sqrt{27} \approx 5.196 \text{ N}$$
$$|\mathbf{q}| = \sqrt{1+16+4} = \sqrt{21} \approx 4.583 \text{ N}$$
$$|\mathbf{r}| = \sqrt{4+25+0} = \sqrt{29} \approx 5.385 \text{ N}$$

$\mathbf{r}$ has the greatest magnitude ($\sqrt{29} \approx 5.39$ N).

**Part (ii): Acceleration of particle of mass 0.4 kg under p, q, r and weight**

Weight (using $g = 10$): $\begin{pmatrix}0\\0\\-0.4\times 10\end{pmatrix} = \begin{pmatrix}0\\0\\-4\end{pmatrix}$ N

Total force:

$$\mathbf{p}+\mathbf{q}+\mathbf{r}+\text{weight} = \begin{pmatrix}-1-1+2\\-1-4+5\\5+2+0-4\end{pmatrix} = \begin{pmatrix}0\\0\\3\end{pmatrix} \text{ N}$$

$$\mathbf{a} = \frac{\begin{pmatrix}0\\0\\3\end{pmatrix}}{0.4} = \begin{pmatrix}0\\0\\7.5\end{pmatrix} \text{ m s}^{-2}$$

Magnitude of acceleration $= 7.5$ m s$^{-2}$, directed **vertically upwards**.

*End of Worked Solutions*