

Constant Acceleration in Two Dimensions (From Edexcel 6677)

Q1, (Jun 2010, Q1)

A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$. At time $t = 6 \text{ s}$ P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j}) \text{ m}$. Find the distance of P from the origin at time $t = 2 \text{ s}$.

(5)

Q2, (Jan 2009, Q1)

A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j}) \text{ m s}^{-2}$. At time $t = 0$, P has speed $u \text{ m s}^{-1}$. At time $t = 3 \text{ s}$, P has velocity $(-6\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$.

Find the value of u .

(5)

Q3, (Jan 2005, Q7)

Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\mathbf{i} + 10\mathbf{j}) \text{ km}$ relative to a fixed origin O . At the same time, Q is at the point with position vector $(14\mathbf{i} - 6\mathbf{j}) \text{ km}$. Three hours later, P is at the point with position vector $(29\mathbf{i} + 34\mathbf{j}) \text{ km}$. The ship Q travels with velocity $12\mathbf{j} \text{ km h}^{-1}$. At time t hours after midnight, the position vectors of P and Q are $\mathbf{p} \text{ km}$ and $\mathbf{q} \text{ km}$ respectively. Find

(a) the velocity of P , in terms of \mathbf{i} and \mathbf{j} ,

(2)

(b) expressions for \mathbf{p} and \mathbf{q} , in terms of t , \mathbf{i} and \mathbf{j} .

(4)

At time t hours after midnight, the distance between P and Q is $d \text{ km}$.

(c) By finding an expression for \overrightarrow{PQ} , show that

$$d^2 = 25t^2 - 92t + 292.$$

(5)

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into sight of the observer,

(d) find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

(5)

Q4, (Jun 2006, Q7)

[In this question the unit vectors \mathbf{i} and \mathbf{j} are due east and north respectively.]

A ship S is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j}) \text{ km h}^{-1}$. At time 1200, the position vector of S relative to a fixed origin O is $(16\mathbf{i} + 5\mathbf{j}) \text{ km}$. Find

(a) the speed of S , (2)

(b) the bearing on which S is moving. (2)

The ship is heading directly towards a submerged rock R . A radar tracking station calculates that, if S continues on the same course with the same speed, it will hit R at the time 1500.

(c) Find the position vector of R . (2)

The tracking station warns the ship's captain of the situation. The captain maintains S on its course with the same speed until the time is 1400. He then changes course so that S moves due north at a constant speed of 5 km h^{-1} . Assuming that S continues to move with this new constant velocity, find

(d) an expression for the position vector of the ship t hours after 1400, (4)

(e) the time when S will be due east of R , (2)

(f) the distance of S from R at the time 1600. (3)

Q5, (Jan 2011, Q4)

A particle P of mass 2 kg is moving under the action of a constant force \mathbf{F} newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j}) \text{ m s}^{-1}$ at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$ at time $t = 5 \text{ s}$.

Find

- (a) the speed of P at $t = 0$, (2)
- (b) the vector \mathbf{F} in the form $a\mathbf{i} + b\mathbf{j}$, (5)
- (c) the value of t when P is moving parallel to \mathbf{i} . (4)

Q6, (Jun 2007, Q7)

A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j}) \text{ km}$ with respect to a fixed origin O . At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j}) \text{ km}$.

- (a) Find the velocity of B , giving your answer in the form $p\mathbf{i} + q\mathbf{j}$. (3)

At time t hours after noon, the position vector of B is $\mathbf{b} \text{ km}$.

- (b) Find, in terms of t , an expression for \mathbf{b} . (3)

Another boat C is also moving with constant velocity. The position vector of C , $\mathbf{c} \text{ km}$, at time t hours after noon, is given by

$$\mathbf{c} = (-9\mathbf{i} + 20\mathbf{j}) + t(6\mathbf{i} + \lambda\mathbf{j}),$$

where λ is a constant. Given that C intercepts B ,

- (c) find the value of λ , (5)
- (d) show that, before C intercepts B , the boats are moving with the same speed. (3)

Q7, (Jun 2013(R), Q6)

[In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O .]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j}) \text{ km}$.

(a) Find the position vector of S at time t hours. (2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j}) \text{ km h}^{-1}$. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j}) \text{ km}$. The two ships meet at the point P .

(b) Find the value of n . (5)

(c) Find the distance OP . (4)

Q8, (Jun 2013, Q7)

[In this question, the horizontal unit vectors \mathbf{i} and \mathbf{j} are directed due east and due north respectively.]

The velocity, $\mathbf{v} \text{ m s}^{-1}$, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$ (3)

(b) Find the bearing on which P is moving when $t = 2$ (2)

(c) Find the value of t when P is moving

(i) parallel to \mathbf{j} ,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)