Constant Acceleration in Two Dimensions (Sheet 2)

Q1.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for question = 10 marks)

Q2.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, P and Q, are moving with constant velocities.
The velocity of P is $(9\mathbf{i} - 2\mathbf{j})$km h^{-1} and the velocity of Q is $(4\mathbf{i} + 8\mathbf{j})$ km h^{-1}

(a) Find the direction of motion of P, giving your answer as a bearing to the nearest degree.

(3)

When $t = 0$, the position vector of P is $(9\mathbf{i} + 10\mathbf{j})$km and the position vector of Q is $(\mathbf{i} + 4\mathbf{j})$ km. At time t hours, the position vectors of P and Q are **p** km and **q** km respectively.

(b) Find an expression for

(i) **p** in terms of t,
(ii) **q** in terms of t.

(3)

(c) Hence show that, at time t hours,

$\overrightarrow{QP} = (8 + 5t)\mathbf{i} + (6 - 10t)\mathbf{j}$

(2)

(d) Find the values of t when the ships are 10 km apart.

(6)

(Total for question = 14 marks)

Q3.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is $(15\mathbf{i} + 20\mathbf{j})$ m s^{-1} and the velocity of Q is $(20\mathbf{i} - 5\mathbf{j})$ m s^{-1}

(a) Find the direction of motion of Q, giving your answer as a bearing to the nearest degree.

(3)

At time $t = 0$, the position vector of P is $400\mathbf{i}$ metres and the position vector of Q is $800\mathbf{j}$ metres. At time t seconds, the position vectors of P and Q are $\mathbf{p}$ metres and $\mathbf{q}$ metres respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,
(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Find the position vector of Q when Q is due west of P.

(4)

(Total for question = 10 marks)

Q4.

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle P.

The force $\mathbf{F}_1$ is given by $\mathbf{F}_1 = (-\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + \mathbf{j})$.

Given that the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + 3\mathbf{j})$,

(a) find $\mathbf{F}_2$

(7)

The acceleration of P is $(3\mathbf{i} + 9\mathbf{j})$ m s^{-2}. At time $t = 0$, the velocity of P is $(3\mathbf{i} - 22\mathbf{j})$ m s^{-1}

(b) Find the speed of P when $t = 3$ seconds.

(4)

(Total for question = 11 marks)

Q5.

A particle P of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} - 2\mathbf{j})$ N.

(a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$ m s^{-2}.

(4)

At time $t = 0$, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

(b) Find the velocity of P at time $t = 2$ seconds.

(3)

Another particle Q moves with constant velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j})$ m s^{-1}.

(c) Find the distance moved by Q in 2 seconds.

(2)

(d) Show that at time $t = 3.5$ seconds both particles are moving in the same direction.

(3)

(Total 12 marks)

Q6.

Two forces $(4\mathbf{i} - 2\mathbf{j})$ N and $(2\mathbf{i} + q\mathbf{j})$ N act on a particle P of mass 1.5 kg. The resultant of these two forces is parallel to the vector $(2\mathbf{i} + \mathbf{j})$.

(a) Find the value of q.

(4)

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 4\mathbf{j})$m s^{-1}.

(b) Find the speed of P at time $t = 2$ seconds.

(6)

(Total 10 marks)

Q7.

A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.
Find the value of u.

(5)

(Total 5 marks)

Q8.

Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is **R**. Given that **R** acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,
(a) find the angle between **R** and the vector **j**,

(3)

(b) show that $2p + q + 3 = 0$.

(4)

Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},
(c) find the value of m.

(7)

(Total 14 marks)

Q9.

A particle P of mass 0.4 kg moves under the action of a single constant force **F** newtons. The acceleration of P is $(6\mathbf{i} + 8\mathbf{j})$ m s^{-2} . Find

(a) the angle between the acceleration and **i**,

(2)

(b) the magnitude of **F**.

(3)

At time t seconds the velocity of P is **v** m s^{-1} . Given that when $t = 0$, $\mathbf{v} = 9\mathbf{i} - 10\mathbf{j}$,

(c) find the velocity of P when $t = 5$.

(3)

(Total 8 marks)