Constant Acceleration in Two Dimensions (Sheet 2) Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 ft	1.1b
		(3)	
			(10 marks)

Notes:

(a)
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
A1: for given answer correctly obtained

(b)
M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
A1: for a correct expression for **r** in terms of t

(c)
M1: for equating the **i** and **j** components of their **r**
A1ft: for a correct equation following their **r**
A1: for $t = 1.5$

(d)
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t
M1: for equating the **i** and **j** components of their **v**
A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

Q2.

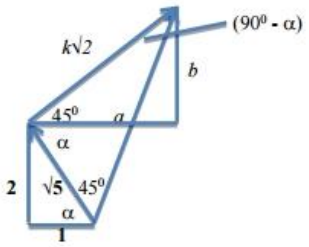
Question Number	Scheme	Marks
(a)	$\tan\theta = \frac{2}{9}$ $\theta = 12.5°$ bearing $103°$	M1 A1 A1 (3)
(b) **(i)** **(ii)**	$\mathbf{p} = (9\mathbf{i}+10\mathbf{j})+t(9\mathbf{i}-2\mathbf{j})$ $\mathbf{q} = (\mathbf{i}+4\mathbf{j})+t(4\mathbf{i}+8\mathbf{j})$	M1 A1 A1 (3)
(c)	$\overrightarrow{QP} = (8+5t)\mathbf{i}+(6-10t)\mathbf{j}$	M1 A1 (2)
(d)	$D^2 = (8+5t)^2+(6-10t)^2$ $= 125t^2-40t+100$ $100 = 125t^2-40t+100$ $0 = 5t(25t-8)$ $t = 0$ or 0.32	M1 A1 M1 M1 A1 A1 (6) **14**
	Notes	
(a)	M1 for $\tan\theta = \pm\frac{2}{9}$ or $\pm\frac{9}{2}$ or use $\sin\theta$ or $\cos\theta$	
	First A1 for $\theta = \pm 13°$ or $\pm 77°$ or $\pm 12.5°$ or $\pm 77.5°$ or better	
	Second A1 for $103°$	
(b)	M1 for clear attempt at $\mathbf{p} = (9\mathbf{i}+10\mathbf{j})+t(9\mathbf{i}-2\mathbf{j})$ or $\mathbf{q} = (\mathbf{i}+4\mathbf{j})+t(4\mathbf{i}+8\mathbf{j})$ (Allow slips but must be a '+' sign and $\mathbf{r}+t\mathbf{v}$)	
(i)	First A1 for $\mathbf{p} = (9\mathbf{i}+10\mathbf{j})+t(9\mathbf{i}-2\mathbf{j})$oe	
(ii)	Second A1 for $\mathbf{q} = (\mathbf{i}+4\mathbf{j})+t(4\mathbf{i}+8\mathbf{j})$oe	
(c)	M1 for $\mathbf{p}-\mathbf{q}$ or $\mathbf{q}-\mathbf{p}$ with their $\mathbf{p}$ and $\mathbf{q}$ substituted A1 for correct answer $\overrightarrow{QP} = (8+5t)\mathbf{i}+(6-10t)\mathbf{j}$ (don't need $\overrightarrow{QP}$ but on R.H.S must be **identical** coefficients of $\mathbf{i}$ and $\mathbf{j}$ but allow column vectors)	
(d)	First M1 for attempt to find QP or QP^2 in terms of t *only*, using correct formula First A1 for a correct expression (with or without √) $125t^2-40t+100$ Second M1 for √ (3 term quadratic) = 10 or (3 term quadratic) =100. Third M1 for quadratic expression = 0 and attempt to solve (e.g. factorising or using formula) Second A1 for $t = 0$ (if they divide by t and lose this value but get 0.32, M1A0A1) Third A1 for $t = 0.32$ oe	

Q3.

Question Number	Scheme	Marks
(a)	$\tan\theta = \frac{5}{20}$ $\theta = 14.036..^0$ $\theta = 104^0$ nearest degree	M1 A1 A1 (3)
(b)	$\mathbf{p} = 400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ $\mathbf{q} = 800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$	M1 A1 A1 (3)
(c)	Equate their **j** components: $20t(\mathbf{j}) = (800 - 5t)(\mathbf{j})$ $t = 32$ $\mathbf{s} = 800\mathbf{j} + 32(20\mathbf{i} - 5\mathbf{j})$ $= 640\mathbf{i} + 640\mathbf{j}$	M1 A1 M1 A1 (4) **10**
(a)	**Notes** **Allow column vectors throughout** M1 for $\tan\theta = \pm\frac{5}{20}$ or $\pm\frac{20}{5}$ (or any other complete method) First A1 for $\pm 14.04°$ or $\pm 75.96°$ Second A1 for $104°$	
(b) **(i)** **(ii)**	M1 for clear attempt at either **p** or **q** (allow slip but t must be attached to the velocity vector and position vector and velocity vector must be paired up correctly) First A1 $400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ "**p** =" not needed but must be clear it's P Second A1 $800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$ "**q** =" not needed but must be clear it's Q	
(c)	First M1 for equating their **j** components; allow **j**'s on both sides First A1 for $t = 32$ Second M1 independent for substituting their t value into their **q** from (b) Second A1 for $640\mathbf{i} + 640\mathbf{j}$	

Q4.

Question Number	Scheme	Marks
(a)	$\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$ $(-1+a)\mathbf{i} + (2+b)\mathbf{j}$ $\frac{-1+a}{2+b} = \frac{1}{3}$ $a = b = k = 2.5;\ \mathbf{F}_2 = 2.5\mathbf{i} + 2.5\mathbf{j}$	B1 M1 **DM1** A1 **DM1** A1; A1 (7)
	ALTERNATIVE: $\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$ $(-1+a)\mathbf{i} + (2+b)\mathbf{j} = p(\mathbf{i} + 3\mathbf{j})$ $-1+a = p$ $2+b = 3p$ $a = b = k = 2.5;\ \mathbf{F}_2 = 2.5\mathbf{i} + 2.5\mathbf{j}$	B1 M1 for LHS **DM1** A1 **DM1** A1; A1 (7)
(b)	$\mathbf{v} = 3\mathbf{i} - 22\mathbf{j} + 3(3\mathbf{i} + 9\mathbf{j})$ $= 12\mathbf{i} + 5\mathbf{j}$ $\lvert\mathbf{v}\rvert = \sqrt{12^2 + 5^2} = 13\ \text{ms}^{-1}$	M1 A1 M1 A1 **cso** (4) **11**
	Notes	
(a)	B1 for $\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$ $(k \neq 1)$ seen or implied in working, including for an incorrect final answer, with the wrong k value. First M1 for adding the 2 forces (for this M mark we only need $\mathbf{F}_2 = a\mathbf{i} + b\mathbf{j}$), with $\mathbf{i}$'s and $\mathbf{j}$'s collected (which can be implied by later working) but allow a slip. (M0 if a and b both assumed to be 1) Second M1, dependent on first M1, for ratio of their cpts = 1/3 or 3/1 (Must be correct way up for the M mark) First A1 for a correct equation which may involve two unknowns Third M1, dependent on first and second M1, for solving for k oe Second A1 for a correct k value Third A1 for $2.5\mathbf{i} + 2.5\mathbf{j}$	

	ALTERNATIVE: Using two simultaneous equations B1 for $\mathbf{F}_2 = k\mathbf{i}+k\mathbf{j}$ ($k \neq 1$) seen or implied in working. First M1 for adding the 2 forces (for this M mark we only need $\mathbf{F}_2 = a\mathbf{i}+b\mathbf{j}$), with **i**'s and **j**'s collected (LHS of equation) (M0 if *a* and *b* both assumed to be 1) but allow a slip Second M1, dependent on first M1, for equating coeffs to produce *two* equations in 2 or 3 unknowns. Must have p and $3p$ (M0 if p is assumed to be 1 or k) First A1 for two correct equations Third M1, dependent on first and second M1, for solving for k oe Second A1 for a correct k value Third A1 for $2.5\mathbf{i}+2.5\mathbf{j}$ **ALTERNATIVE: Using magnitudes and directions**  $\mathbf{F}_2 = k\mathbf{i}+k\mathbf{j}$, seen or implied Correct vector triangle $\frac{k\sqrt{2}}{\sin 45^0} = \frac{\sqrt{5}}{\sin(90^0-\alpha)}$, $\alpha = \arctan 2$ $2k = 5$ $k = 2.5$; $\mathbf{F}_2 = 2.5\mathbf{i}+2.5\mathbf{j}$	B1 M1 **DM1** A1 **DM1** A1; A1 (7)

	ALTERNATIVE: Using magnitudes and directions B1 for $\mathbf{F}_2 = k\mathbf{i}+k\mathbf{j}$ seen or implied in working. First M1 for a correct vector triangle (for this M mark we only need $\mathbf{F}_2 = a\mathbf{i}+b\mathbf{j}$). (M0 if *a* and *b* both assumed to be 1 and/or longest side is assumed to be $\sqrt{10}$) Second M1, dependent on first M1, for using sine rule on vector triangle First A1 for a correct equation. 45^0 may not appear exactly. Third M1, dependent on first and second M1, for solving for k oe Second A1 for a correct k value Third A1 for $2.5\mathbf{i}+2.5\mathbf{j}$	

(b)	First M1 for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 3$ First A1 for $12\mathbf{i}+5\mathbf{j}$ seen or implied. However, if a wrong **v** is seen A0 Second M1 for finding magnitude of their **v** Second A1 for 13	

Q5.

Question Number	Scheme	Marks
a	$\mathbf{F} = m\mathbf{a}$: $3\mathbf{i} - 2\mathbf{j} = 0.5\mathbf{a}$ $\mathbf{a} = 6\mathbf{i} - 4\mathbf{j}$ $\lvert\mathbf{a}\rvert = \sqrt{6^2 + (-4)^2} = 2\sqrt{13}\ (\text{m s}^{-2})$ **	M1 A1 M1A1 (4)
b	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = (\mathbf{i} + 3\mathbf{j}) + 2(6\mathbf{i} - 4\mathbf{j})$	M1A1 ft
	$= 13\mathbf{i} - 5\mathbf{j}\ \text{m s}^{-1}$	A1 (3)
c	Distance $= 2\lvert\mathbf{v}\rvert = 2\sqrt{4+1} = 2\sqrt{5} = 4.47\ (\text{m})$	M1A1
		(2)
d	When $t = 3.5$, velocity of P is $(\mathbf{i} + 3\mathbf{j}) + 3.5(6\mathbf{i} - 4\mathbf{j}) = 22\mathbf{i} - 11\mathbf{j}$	M1A1 ft
	Given conclusion reached correctly. E.g. $22\mathbf{i} - 11\mathbf{j} = 11(2\mathbf{i} - \mathbf{j})$	A1 (3)
		[12]

Notes for Question

Question (a)

Either:
First M1 for use of $\mathbf{F} = m\,\mathbf{a}$
First A1 for $\mathbf{a} = 6\mathbf{i} - 4\mathbf{j}$
Second M1 for $a = \sqrt{(6^2 + (-4)^2)}$ (Allow $\sqrt{(6^2 + 4^2)}$)
Second A1 for $a = 2\sqrt{13}\ (\text{ms}^{-2})$ **Given answer**

Or:
First M1 for $F = \sqrt{(3^2 + (-2)^2)}$ (Allow $\sqrt{(3^2 + 2^2)}$)
First A1 $F = \sqrt{13}$
Second M1 for $\sqrt{13} = 0.5\,a$
Second A1 for $a = 2\sqrt{13}\ (\text{ms}^{-2})$ **Given answer**

Question (b)
M1 for $(\mathbf{i} + 3\mathbf{j}) + (2 \times \text{their } \mathbf{a})$
First A1 **ft** for a correct expression
Second A1 for $13\mathbf{i} - 5\mathbf{j}$; isw if they go on to find the speed

Question (c)
M1 for $2\sqrt{(2^2 + (-1)^2)}$ or $\sqrt{(4^2 + (-2)^2)}$
A1 for $2\sqrt{5}$ or $\sqrt{20}$ or 4.5 or 4.47 or better

Question (d)
M1 for $(\mathbf{i} + 3\mathbf{j}) + (3.5 \times \text{their } \mathbf{a})$,or possibly, their (b) + (1.5 x their **a**)
First A1 ft for a correct expression *of form* $a\mathbf{i} + b\mathbf{j}$
Second A1 for given conclusion reached correctly *e.g.* $22\mathbf{i} - 11\mathbf{j} = 11(2\mathbf{i} - \mathbf{j})$ oe **Given answer**

Q6.

Question Number	Scheme	Marks
(a)	$(4\mathbf{i}-2\mathbf{j})+(2\mathbf{i}+q\mathbf{j}) = (6\mathbf{i}+(q-2)\mathbf{j})$ $6 = 2(q-2)$ ratio 2:1 $q = 5$	M1A1 **DM1** A1 (4)
(b)	$6\mathbf{i}+3\mathbf{j} = 1.5\mathbf{a}$ $\mathbf{a} = (4\mathbf{i}+2\mathbf{j})$ m s^{-2} $\mathbf{v} = \mathbf{u}+\mathbf{at} = (-2\mathbf{i}+4\mathbf{j})+2(4\mathbf{i}+2\mathbf{j})$ $= 6\mathbf{i}+8\mathbf{j}$ speed $= \sqrt{6^2+8^2}$ $= 10$ m s^{-1}	M1 A1 M1 A1ft **M1** A1 (6) **[10]**

Notes for Question

Question (a)
First M1 for $(4\mathbf{i}-2\mathbf{j})+(2\mathbf{i}+q\mathbf{j})$
First A1 for $(6\mathbf{i}+(q-2)\mathbf{j})$ (seen or implied)
Second M1, **dependent on first M1**, for using 'parallel to $(2\mathbf{i}+\mathbf{j})$' to obtain an equation in q *only*.
Second A1 for $q = 5$

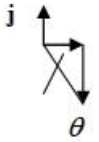
Question (b)
First M1 for their **resultant force** $= 1.5\mathbf{a}$
First A1 for $\mathbf{a} = 4\mathbf{i}+2\mathbf{j}$
Second M1 for $(-2\mathbf{i}+4\mathbf{j})+2$ x (their **a**) **(M0 if force is used instead of a)**
Second A1 **ft** for their velocity at $t = 2$
Third M1for finding the magnitude of their velocity at $t = 2$
Third A1 for 10 (ms^{-1})

N.B. In (b), if they use scalars throughout, M0A0M0A0M0A0

Q7.

Question Number	Scheme	Marks
	$-6\mathbf{i}+\mathbf{j} = \mathbf{u}+3(2\mathbf{i}-5\mathbf{j})$ $\Rightarrow \mathbf{u} = -12\mathbf{i}+16\mathbf{j}$ $\Rightarrow u = \sqrt{(-12)^2+16^2} = 20$	M1 A1 A1 cso M1 A1 **[5]**

Q8.

Question Number	Scheme	Marks
(a)	 $\tan\theta = \frac{2}{1} \Rightarrow \theta = 63.4^{\circ}$	M1 A1
	angle is 153.4°	A1 (3)
(b)	$(4+p)\mathbf{i} + (q-5)\mathbf{j}$	B1
	$(q-5) = -2(4+p)$	M1 A1
	$2p + q + 3 = 0$ *	A1 (4)
(c)	$q = 1 \Rightarrow p = -2$	B1
	$\Rightarrow \mathbf{R} = 2\mathbf{i} - 4\mathbf{j}$	M1
	$\Rightarrow \lvert\mathbf{R}\rvert = \sqrt{2^2 + (-4)^2} = \sqrt{20}$	M1 A1 f.t.
	$\sqrt{20} = m8\sqrt{5}$	M1 A1 f.t.
	$\Rightarrow m = \frac{1}{4}$	A1 cao (7)
		[14]

Q9.

Question Number	Scheme	Marks
(a)	$\tan\theta = \frac{8}{6}$	M1
	$\theta \approx 53°$	A1 (2)
(b)	$\mathbf{F} = 0.4(6\mathbf{i} + 8\mathbf{j})$ $(= 2.4\mathbf{i} + 3.2\mathbf{j})$	M1
	$\lvert\mathbf{F}\rvert = \sqrt{(2.4^2 + 3.2^2)} = 4$	M1 A1 (3)
(c)	$\mathbf{v} = 9\mathbf{i} - 10\mathbf{j} + 5(6\mathbf{i} + 8\mathbf{j})$	M1 A1
	$= 39\mathbf{i} + 30\mathbf{j}$ (ms^{-1})	A1 (3)
		(8 marks)