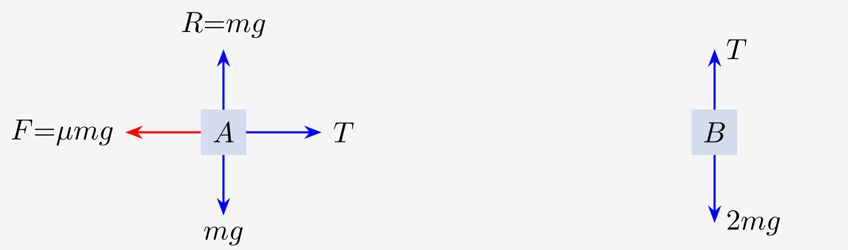
Question 1

Worked Solution

Force diagrams



Particle A (moves $\rightarrow$) Particle B (moves $\downarrow$)

Part (a): Tension immediately after release

Acceleration $a = \frac{4}{9}g$ (given). For particle B (downward positive):

$$2mg - T = 2m \cdot \frac{4g}{9} \implies T = 2mg - \frac{8mg}{9}$$

$$T = \frac{10mg}{9}$$

Part (b): Show that $\mu = \dfrac{2}{3}$

For particle A (towards pulley positive):

$$\frac{10mg}{9} - \mu mg = \frac{4mg}{9} \implies \mu mg = \frac{6mg}{9}$$

$$\mu = \frac{2}{3} \quad \checkmark$$

Part (c): Speed of A as it reaches P

Phase 1 – B falls height h:

$$v_1^2 = 2 \cdot \frac{4g}{9} \cdot h = \frac{8gh}{9}$$

At this moment A is $\frac{1}{3}h$ from P.

Phase 2 – string slack, A decelerates under friction:

$$a' = \mu g = \frac{2g}{3}, \qquad V^2 = \frac{8gh}{9} - 2 \cdot \frac{2g}{3} \cdot \frac{h}{3} = \frac{8gh}{9} - \frac{4gh}{9} = \frac{4gh}{9}$$

$$V = \frac{2}{3}\sqrt{gh}$$

Part (d): Use of “light string”

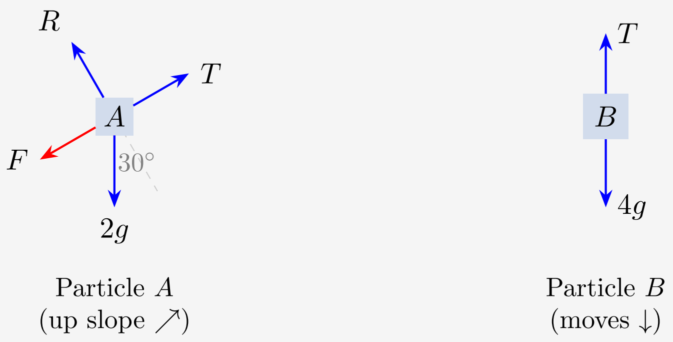
The tension is the same throughout the string, so the same tension T acts on both A and B.

Question 2

Worked Solution

$\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$. $\mu = \frac{1}{\sqrt{3}}$. A (2 kg) moves *up* the plane; B (4 kg) moves *down*.

Force diagrams



Tension immediately after release

Perpendicular to plane: $R = 2g\cos 30° = g\sqrt{3}$

Friction (down slope, A moves up): $F = \frac{1}{\sqrt{3}} \cdot g\sqrt{3} = g$

Equation of motion for B (downward positive):

$$4g - T = 4a \quad \cdots (1)$$

Equation of motion for A (up slope positive):

$$T - 2g\sin 30° - F = 2a \implies T - g - g = 2a \implies T - 2g = 2a \quad \cdots (2)$$

(1) + (2): $2g = 6a \implies a = \frac{g}{3}$. From (1): $T = 4g - \frac{4g}{3} = \frac{8g}{3}$

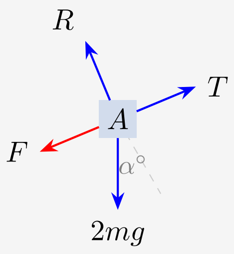
$$T = \frac{8g}{3} \approx 26.1\,\text{N}$$

Question 3

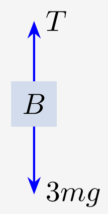
Worked Solution

$\tan\alpha = \frac{5}{12}$, $\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. $\mu = \frac{2}{3}$. A $(2m)$ moves *up*; B $(3m)$ moves *down*.

Force diagrams



Particle A
(up slope ↗)



Particle B
(moves ↓)

Part (a): Show that $T = \dfrac{12mg}{5}$

Perpendicular to plane: $R = 2mg\cos\alpha = \dfrac{24mg}{13}$

Friction: $F = \frac{2}{3}\cdot\dfrac{24mg}{13} = \dfrac{16mg}{13}$

Equation of motion for A (up slope positive):

$$T - \frac{16mg}{13} - \frac{10mg}{13} = 2ma \implies T - 2mg = 2ma \quad \cdots(1)$$

Equation of motion for B (downward positive):

$$3mg - T = 3ma \quad \cdots(2)$$

(1) + (2): $mg = 5ma \implies a = \dfrac{g}{5}$. From (2): $T = 3mg - \dfrac{3mg}{5} = \dfrac{12mg}{5}$

$$T = \frac{12mg}{5} \quad \checkmark$$

Part (b): Will A remain at rest after B hits the ground?

After A stops, compare forces along slope:

$$\text{Weight component down slope} = 2mg\sin\alpha = \frac{10mg}{13}$$

$$F_{\max} = \frac{16mg}{13} > \frac{10mg}{13}$$

A will remain at rest: the maximum friction force $\left(\frac{16mg}{13}\right)$ exceeds the weight component down the slope $\left(\frac{10mg}{13}\right)$.

Part (c): Two refinements to the model

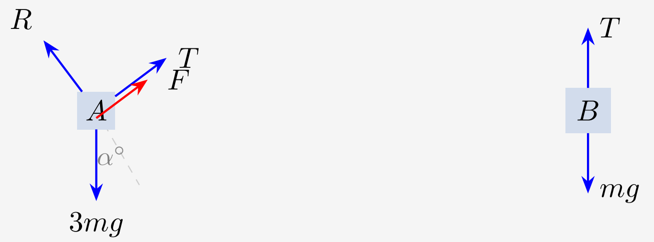
- Account for air resistance (model blocks as non-particles subject to drag).
- Use an extensible string to model the elastic properties of a real string.

Question 4

Worked Solution

$\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$. $\mu = \frac{1}{6}$. A ($3m$) moves *down* the plane; B (m) moves *up*.

Force diagrams



Particle A (down slope ↙)

Particle B (moves ↑)

Part (a): Equation of motion for A

Perpendicular to plane: $R = 3mg\cos\alpha = \dfrac{12mg}{5}$

Friction (up slope, opposing downward motion): $F = \frac{1}{6} \cdot \dfrac{12mg}{5} = \dfrac{2mg}{5}$

Taking down the slope as positive:

$$3mg\sin\alpha - F - T = 3ma \iff \frac{7mg}{5} - T = 3ma$$

Part (b): Show that acceleration is $\dfrac{1}{10}g$

Equation of motion for B (upward positive):

$$T - mg = ma \quad \cdots (2)$$

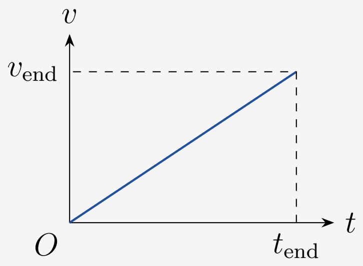
Adding the A equation to (2):

$$\tfrac{7mg}{5} - mg = 4ma \implies \tfrac{2mg}{5} = 4ma \implies a = \tfrac{g}{10}$$

$$a = \frac{1}{10}g \quad \checkmark$$

Part (c): Velocity–time graph for B

All forces are constant, so B accelerates uniformly; the v–t graph is a straight line from the origin.



The acceleration of B is constant, so the graph is a straight line from O with gradient $\frac{g}{10}$.

Part (d): Effect of string not being light

If the string is not light, its mass must be included. The tension would not be constant throughout: the tension on A would differ from the tension on B, so two separate tension values would appear in the equations of motion.

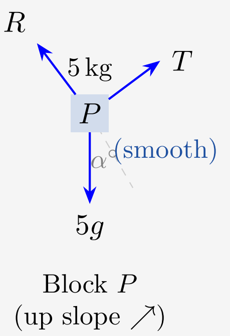
Question 5

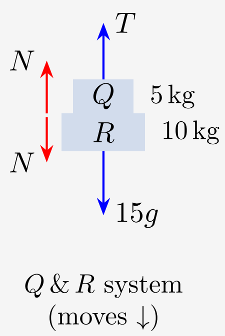
Worked Solution

$\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$. Plane is **smooth**.

P: 5 kg on smooth slope (moves up). Scale pan (light) carries Q (5 kg) on top of R (10 kg); total hanging mass = 15 kg (moves down).

Force diagrams





Part (a): Acceleration and tension

Hanging side (downward positive):

$$15g - T = 15a \quad \cdots (1)$$

Block P (up slope positive, smooth):

$$T - 5g\sin\alpha = 5a \implies T - 3g = 5a \quad \cdots (2)$$

(1) + (2): $12g = 20a \implies a = 0.6g$. From (2): $T = 3g + 3g = 6g$

(i) $a = 0.6g \approx 5.88\,\text{m}\,\text{s}^{-2}$ (ii) $T = 6g = 58.8\,\text{N}$

Part (b): Force exerted on Q by R

Consider Q alone (5 kg, accelerating downward at $0.6g$):

$$5g - N = 5 \times 0.6g = 3g \implies N = 2g$$

Force on Q from $R = 2g \approx 19.6\,\text{N}$

Part (c): Force on the pulley by the string

Tension $T = 6g$ acts on each side of the pulley. One side is vertical (toward scale pan); the other is directed down the slope at angle α below horizontal. Resolving both tensions into the pulley:

$$\text{Vertical:} \quad 6g + 6g\sin\alpha = 6g\left(1 + \tfrac{3}{5}\right) = \tfrac{48g}{5}$$
$$\text{Horizontal:} \quad 6g\cos\alpha = 6g \cdot \tfrac{4}{5} = \tfrac{24g}{5}$$

$$|\mathbf{F}| = \sqrt{\left(\tfrac{48g}{5}\right)^2 + \left(\tfrac{24g}{5}\right)^2} = \tfrac{24g}{5}\sqrt{5} = \tfrac{24g\sqrt{5}}{5} \approx 105\,\text{N}$$

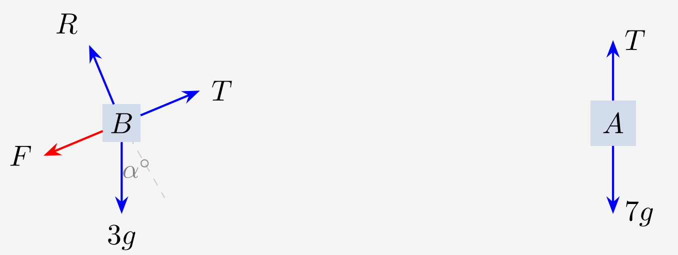
Force on pulley $= \dfrac{24g\sqrt{5}}{5} \approx 105\,\text{N}$

Question 6

Worked Solution

$\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$. A: 7 kg hanging (moves down); B: 3 kg on rough plane, $\mu = \frac{2}{3}$ (moves up).

Force diagrams



Particle B (up slope ↗)

Particle A (moves ↓)

Part (a): Acceleration of B immediately after release

Perpendicular to plane: $R = 3g\cos\theta = \dfrac{36g}{13}$

Friction: $F = \frac{2}{3} \cdot \dfrac{36g}{13} = \dfrac{24g}{13}$

For A (downward positive): $7g - T = 7a \quad \cdots (1)$

For B (up slope positive): $T - \frac{24g}{13} - \frac{15g}{13} = 3a \implies T - 3g = 3a \quad \cdots (2)$

(1) + (2): $4g = 10a \implies a = \dfrac{2g}{5}$

Acceleration $= \dfrac{2g}{5} = 3.92\,\text{m}\,\text{s}^{-2}$

Part (b): Speed of B after moving 1 m up the plane

$v^2 = 2 \times \dfrac{2 \times 9.8}{5} \times 1 = 7.84$

$v = 2.8\,\text{m}\,\text{s}^{-1}$

Part (c): Time from string breaking to B coming to rest

After the string breaks, both friction and gravity component act down the slope:

$$3a = -\left(\tfrac{24g}{13} + \tfrac{15g}{13}\right) = -3g \implies a = -g$$

$0 = 2.8 - 9.8t \implies t = \dfrac{2.8}{9.8} = \dfrac{2}{7}$

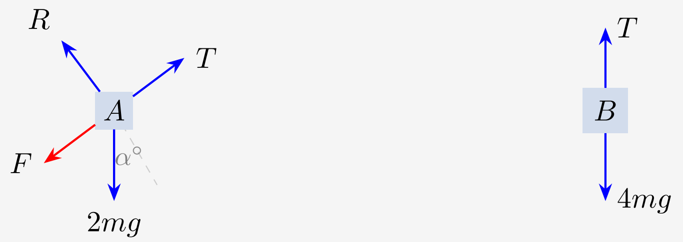
$$t = \frac{2}{7} \approx 0.286\,\text{s}$$

Question 7

Worked Solution

$\tan a = \frac{3}{4}$, $\sin a = \frac{3}{5}$, $\cos a = \frac{4}{5}$. A: mass $2m$ on rough plane, $\mu = \frac{1}{4}$ (moves up). B: mass $4m$ hanging (moves down).

Force diagrams



Particle A (up slope ↗)   Particle B (moves ↓)

Part (a): Reason accelerations are equal in magnitude

The string is inextensible; its total length is constant. Any increase on one side of the pulley equals the decrease on the other, so both particles move at the same speed and have the same magnitude of acceleration.

Part (b): Equations of motion

Normal reaction: $R = 2mg\cos a = 1.6mg$

Friction (down slope, A moves up): $F = \frac{1}{4} \times 1.6mg = 0.4mg$

For B (downward positive): $4mg - T = 4ma \quad \cdots (1)$

For A (up slope positive): $T - 1.2mg - 0.4mg = 2ma \implies T - 1.6mg = 2ma \quad \cdots (2)$

For B: $4mg - T = 4ma$ For A: $T - 1.6mg = 2ma$

Part (c): Acceleration

$(1) + (2)$: $2.4mg = 6ma \implies a = 0.4g$

$a = 0.4g = 3.92\,\mathrm{m\,s^{-2}}$

Part (d): Distance XY in terms of h

Phase 1: B falls h from rest at $a = 0.4g$; speed when B hits: $v^2 = 2 \times 0.4g \times h = 0.8gh$. A has moved h up the slope.

Phase 2: String slack; gravity component *and* friction now both act down slope:

$$a' = g\sin a + \mu g\cos a = 0.6g + \frac{1}{4} \times 0.8g = 0.8g$$

Additional distance s: $0 = 0.8gh - 2 \times 0.8g \times s \implies s = 0.5h$

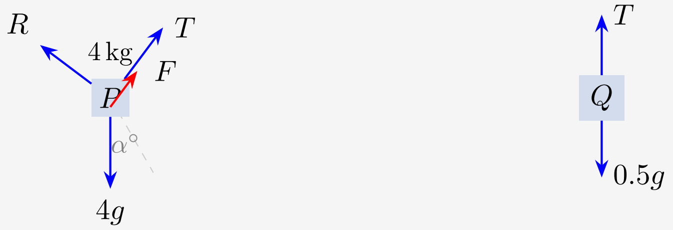
$$XY = h + 0.5h = \frac{3h}{2}$$

Question 8

Worked Solution

$\tan\alpha = \frac{4}{3}$, $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$. P: 4 kg on rough plane, $\mu = 0.5$ (slides down). Q: 0.5 kg hanging (moves up).

Force diagrams



Particle P (down slope ↙)

Particle Q (moves ↑)

Part (a): Tension in the string

Perpendicular to plane: $R = 4g\cos\alpha = \dfrac{12g}{5}$

Friction (up slope, opposing downward motion): $F = 0.5 \times \dfrac{12g}{5} = \dfrac{6g}{5}$

For P (down slope positive):

$$4g\sin\alpha - T - F = 4a \implies \tfrac{16g}{5} - T - \tfrac{6g}{5} = 4a \implies 2g - T = 4a \quad \cdots(1)$$

For Q (upward positive):

$$T - 0.5g = 0.5a \quad \cdots(2)$$

From (2): $T = 0.5g + 0.5a$. Into (1): $1.5g = 4.5a \implies a = \dfrac{g}{3}$

$$T = 0.5g + \tfrac{g}{6} = \tfrac{2g}{3}$$

$$T = \frac{2g}{3} \approx 6.53\,\text{N}$$

Part (b): Resultant force on the pulley by the string

Tension $T = \dfrac{2g}{3}$ on each side of the pulley: one vertical (toward Q), one along the slope toward P (at angle α below horizontal). Resolving both tensions into the pulley:

$$\text{Vertical:} \quad T + T\sin\alpha = \tfrac{2g}{3}\left(1 + \tfrac{4}{5}\right) = \tfrac{6g}{5}$$
$$\text{Horizontal:} \quad T\cos\alpha = \tfrac{2g}{3} \cdot \tfrac{3}{5} = \tfrac{2g}{5}$$

$$|\mathbf{F}| = \sqrt{\left(\tfrac{6g}{5}\right)^2 + \left(\tfrac{2g}{5}\right)^2} = \tfrac{g}{5}\sqrt{40} = \tfrac{2g\sqrt{10}}{5}$$

Resultant force on pulley $= \dfrac{2g\sqrt{10}}{5} \approx 12.4\,\text{N}$

End of Worked Solutions