## Question 1

### Worked Solution

$\mathrm{P}(X = r) = \dfrac{k}{r(r-1)}$ for $r = 2, 3, 4, 5, 6$.

**Part (i): Show $k = 1.2$ and display the distribution**

Since probabilities must sum to 1:

$$\sum_{r=2}^{6} \frac{k}{r(r-1)} = 1$$

$$k\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right) = 1$$

Finding a common denominator (60):

$$k \times \frac{30 + 10 + 5 + 3 + 2}{60} = k \times \frac{50}{60} = 1 \implies k = \frac{60}{50} = 1.2 \quad \checkmark$$

Using $k = 1.2$, the probability distribution is:

| $r$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $\mathrm{P}(X = r)$ | 0.6 | 0.2 | 0.1 | 0.06 | 0.04 |

$k = 1.2$ ✓ Distribution as in the table above.

## Question 2

**Worked Solution**

$P(X = r) = k + 0.01r^2$ for $r = 1, 2, 3, 4, 5$.

**Part (i): Show $k = 0.09$ and display the distribution**

Sum of probabilities = 1:

$$\sum_{r=1}^{5}(k + 0.01r^2) = 1$$

$$5k + 0.01(1 + 4 + 9 + 16 + 25) = 1$$

$$5k + 0.01 \times 55 = 1 \implies 5k + 0.55 = 1 \implies 5k = 0.45 \implies k = 0.09 \quad \checkmark$$

Probability distribution:

| $r$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = r)$ | 0.10 | 0.13 | 0.18 | 0.25 | 0.34 |

$k = 0.09$ ✓ Distribution as in the table above.

## Question 3

**Worked Solution**

$P(X = r) = k(r^2 - 1)$ for $r = 2, 3, 4, 5$.

**Part (i): Display distribution and find $k$**

Sum of probabilities = 1:

$$k(2^2 - 1) + k(3^2 - 1) + k(4^2 - 1) + k(5^2 - 1) = 1$$

$$k(3 + 8 + 15 + 24) = 1 \implies 50k = 1 \implies k = \frac{1}{50} = 0.02$$

Probability distribution:

| $r$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $P(X = r)$ | 0.06 | 0.16 | 0.30 | 0.48 |

$k = 0.02$ (or $\frac{1}{50}$). Distribution as in the table above.

## Question 4

### Worked Solution

$\mathrm{P}(X = r) = kr(r + 1)$ for $r = 1, 2, 3, 4, 5$.

**Part (i): Show** $k = \frac{1}{70}$

Sum of probabilities $= 1$:

$$k(1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + 5 \cdot 6) = 1$$

$$k(2 + 6 + 12 + 20 + 30) = 1 \implies 70k = 1 \implies k = \frac{1}{70} \quad \checkmark$$

$k = \dfrac{1}{70}$ ✓

# Question 5

## Worked Solution

$P(X = x) = \frac{2x - 1}{36}$ for $x = 1, 2, 3, 4, 5, 6$.

**Part (a): Probability distribution table**

Substitute each value of $x$:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{36}$ | $\frac{3}{36}$ | $\frac{5}{36}$ | $\frac{7}{36}$ | $\frac{9}{36}$ | $\frac{11}{36}$ |

Check: $1 + 3 + 5 + 7 + 9 + 11 = 36$, so probabilities sum to 1. ✓

Distribution as in the table above.

**Part (b):** $P(2 < X \leq 5)$

This includes $X = 3, 4, 5$:

$$P(2 < X \leq 5) = \frac{5}{36} + \frac{7}{36} + \frac{9}{36} = \frac{21}{36} = \frac{7}{12}$$

$P(2 < X \leq 5) = \frac{21}{36} = \frac{7}{12} \approx 0.583$

# Question 6

## Worked Solution

Two fair tetrahedral dice, faces 0, 1, 2, 3. $R$ = score on red die, $B$ = score on blue die. $T = R \times B$.

**Part (a):** $\text{P}(R = 3$ **and** $B = 0)$

Since $R$ and $B$ are independent, each equally likely to show 0, 1, 2, or 3:

$$\text{P}(R = 3 \text{ and } B = 0) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\text{P}(R = 3 \text{ and } B = 0) = \frac{1}{16}$$

**Part (b): Sample space for $T = R \times B$**

| $B \backslash R$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 3 | 0 | 3 | 6 | 9 |
| 2 | 0 | 2 | 4 | 6 |
| 1 | 0 | 1 | 2 | 3 |
| 0 | 0 | 0 | 0 | 0 |

Sample space as in the table above. (All 16 equally likely outcomes.)

**Part (c): Find values of $a$, $b$, $c$, $d$**

From the sample space, count occurrences of each value of $T$ (each out of 16 equally likely outcomes):

- $T = 0$: occurs when $R = 0$ (4 times) or $B = 0$ (4 times) — but $(0, 0)$ counted once, so $4 + 4 - 1 = 7$ times. $\text{P}(T = 0) = \frac{7}{16}$
- $T = 1$: $(R = 1, B = 1)$ only — 1 time. $\text{P}(T = 1) = \frac{1}{16}$
- $T = 2$: $(R = 2, B = 1)$ and $(R = 1, B = 2)$ — 2 times. $\text{P}(T = 2) = \frac{2}{16} = \frac{1}{8}$
- $T = 3$: $(R = 3, B = 1)$ and $(R = 1, B = 3)$ — 2 times. $\text{P}(T = 3) = \frac{2}{16} = \frac{1}{8}$
- $T = 4$: $(R = 2, B = 2)$ — 1 time. $\text{P}(T = 4) = \frac{1}{16}$
- $T = 6$: $(R = 3, B = 2)$ and $(R = 2, B = 3)$ — 2 times. $\text{P}(T = 6) = \frac{2}{16} = \frac{1}{8}$
- $T = 9$: $(R = 3, B = 3)$ — 1 time. $\text{P}(T = 9) = \frac{1}{16}$

Matching to the given table ($t$: 0, 1, 2, 3, 4, 6, 9 with $a$, $b$, 1/8, 1/8, $c$, 1/8, $d$):

$$a = \frac{7}{16}, \quad b = \frac{1}{16}, \quad c = \frac{1}{16}, \quad d = \frac{1}{16}$$

## Question 7

**Worked Solution**

$X$ has distribution: $\mathrm{P}(X = x) = kx$ for $x = 1, 2, 3, 4$.

**Part (a): Show** $k = 0.1$

$$k + 2k + 3k + 4k = 1 \implies 10k = 1 \implies k = 0.1 \quad \checkmark$$

$k = 0.1$ ✓

**Part (e): Show** $\mathrm{P}(X_1 + X_2 = 4) = 0.1$

The pairs $(X_1, X_2)$ that sum to 4 are: $(1, 3)$, $(3, 1)$, $(2, 2)$.

$$\mathrm{P}(X_1 = 1, X_2 = 3) = 0.1 \times 0.3 = 0.03$$
$$\mathrm{P}(X_1 = 3, X_2 = 1) = 0.3 \times 0.1 = 0.03$$
$$\mathrm{P}(X_1 = 2, X_2 = 2) = 0.2 \times 0.2 = 0.04$$
$$\mathrm{P}(X_1 + X_2 = 4) = 0.03 + 0.03 + 0.04 = 0.10 \quad \checkmark$$

$\mathrm{P}(X_1 + X_2 = 4) = 0.03 + 0.03 + 0.04 = 0.10$ ✓

**Part (f): Complete the probability distribution for** $Y = X_1 + X_2$

The missing entries in the table are at $y = 5$ and $y = 8$.

$y = 5$**:** Pairs: $(1, 4)$, $(4, 1)$, $(2, 3)$, $(3, 2)$:

$$= 0.1(0.4) + 0.4(0.1) + 0.2(0.3) + 0.3(0.2) = 0.04 + 0.04 + 0.06 + 0.06 = 0.20$$

$y = 8$**:** Only pair: $(4, 4)$:

$$= 0.4 \times 0.4 = 0.16$$

Check: $0.01 + 0.04 + 0.10 + 0.20 + 0.25 + 0.24 + 0.16 = 1.00$ ✓

| $y$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $\mathrm{P}(Y = y)$ | 0.01 | 0.04 | 0.10 | **0.20** | 0.25 | 0.24 | **0.16** |

$\mathrm{P}(Y = 5) = 0.20, \quad \mathrm{P}(Y = 8) = 0.16$

**Part (g):** $\mathrm{P}(1.5 < X_1 + X_2 \leq 3.5)$

This requires $X_1 + X_2 = 2$ or $X_1 + X_2 = 3$ (the only integer values in $(1.5, 3.5]$):

$$\mathrm{P}(1.5 < Y \leq 3.5) = \mathrm{P}(Y = 2) + \mathrm{P}(Y = 3) = 0.01 + 0.04 = 0.05$$

$\mathrm{P}(1.5 < X_1 + X_2 \leq 3.5) = 0.05$

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**End of Worked Solutions**