Discrete Random Variables (Sheet 2) MS

Q1.

Question	Scheme	Marks	AOs
	Overall method	M1	2.1
	$a+b=2c+0.5$ oe or $a+b=2(1-a-b)$	B1	2.2a
	$a+b+c=0.75$ oe	B1	1.1b
	$3c=0.25$ $\left[c=0.0833...\text{ or }\frac{1}{12}\right]$	M1	1.1b
	P(scoring 2,4 or 4,2 or 3,3) = $2\times"\frac{1}{12}"\times 0.15+0.1^2$	M1	3.1b
	$=0.035$ oe	A1cso	1.1b
		(6)	
			(6 marks)

Notes		
	M1:	A fully correct method with all the required steps. For gaining 2 correct equations with at least one correct(allow if unsimplified). Attempting to solve to find a value of c followed by **correct method** to find the probability
	B1:	Forming a correct equation from the information given in the question
	B1:	A correct equation using the sum of the probabilities equals 1
	M1:	Correct method for solving 2 equations to find c Implied by $c=\frac{1}{12}$
	M1:	Recognising the ways to get a total of 6. Condone missing arrangments or repeats. Do not ignore extras written unless ignored in the calculation. May be implied by $m\times"\frac{1}{12}"\times 0.15+n\times 0.1^2$ where m and n are positive integers
	A1cso:	Cao 0.035, $\frac{7}{200}$ oe

Q2.

	Scheme	Marks	AO
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ or $\frac{1}{600}(60k+30k+20k+15k+12k)=1$	M1	1.1b
	So $k=\frac{600}{137}$ (*)	A1cso	1.1b
		(2)	
(b)	(Cases are:) $D_1=30, D_2=50$ and $D_1=50, D_2=30$ and $D_1=40, D_2=40$	M1	2.1
	$\mathrm{P}(D_1+D_2=80)=\frac{k}{50}\times\frac{k}{30}\times 2 + \left(\frac{k}{40}\right)^2$	M1	3.4
	$=0.0375619\ldots$ awrt **0.0376**	A1	1.1b
		(3)	
(c)	Angles are: $a,\quad a+d,\quad a+2d,\quad a+3d$	M1	3.1a
	$S_4=a+(a+d)+(a+2d)+(a+3d)=360$	M1	2.1
	$2a+3d=180$ (o.e.)	A1	2.2a
	Smallest angle is $a>50$ consider cases:	M1	3.1b
	$d=10$ so $a=75$ or $d=20$ so $a=60$ [$d=30$ gives $a=45$ no good]		
	$\mathrm{P}(D=10 \text{ or } 20)=\frac{3k}{20}=\frac{90}{137}$	A1	1.1b
		(5)	
		(10 marks)	

	Notes
(a)	M1 for clear use of sum of probabilities = 1 (all terms seen)
	A1 cso (*) M1 scored and no incorrect working seen.
Verify	**(Assume $k=\frac{600}{137}$)** to score the final A1 they must have a final comment "$\therefore k=\frac{600}{137}$"
(b)	1st M1 for selecting at least 2 of the relevant cases (may be implied by their correct probs) e.g. allow 30, 50 and 50,30 i.e. D_1 and D_2 labels not required
	2nd M1 for using the model to obtain a correct expression for two different probabilities. May use letter k or their value for k.
	Allow for $\frac{k}{50}\times\frac{k}{30}+\left(\frac{k}{40}\right)^2$ or $2\times\left(\frac{k}{50}\times\frac{k}{30}+\left(\frac{k}{40}\right)^2\right)$
	A1 for awrt 0.0376 (exact fraction is $\frac{705}{18769}$)
(c)	1st M1 for recognising the 4 angles and finding expressions in terms of d and their a
	2nd M1 for using property of quad with these 4 angles (equation can be un-simplified) Allow these two marks for use of a (possible) value of d
	e.g. $a+a+10+a+20+a+30=360$ (If at least 3 cases seen allow A1 for e.g. $4a=300$)
	or allow M1M1 for a set of 4 angles with sum 360 and possible value of d (3 cases for A1)
	e.g. (for $d=20$) 60, 80, 100, 120
	1st A1 for $2a+3d=180$ condition (o.e.) [Must be in the form $pa+qd=N$]
	3rd M1 for examining cases and getting $d=10$ and $d=20$ only
	2nd A1 for $\frac{90}{137}$ or exact equivalent
	The correct answer and no obviously incorrect working will score 5/5
	A final answer of awrt 0.657 (0.65693…) with no obviously incorrect working scores 4/5

Q3.

Question Number	Scheme	Marks
(a)	$0.4+p+0.05+0.15+p=1$ <u>or</u> verify $0.4+0.2+0.05+0.15+0.2=1$ $2p=0.4$ $p=0.2$ conclusion $p=0.2$ must be stated	M1 A1cso **(2)**
(b)	$E(X)=0.4\times-4+0.2\times-2+0.05\times1+0.15\times3+0.2\times5=-0.5$	M1 A1 **(2)**
(c)	$[F(0)=P(X=-2)+P(X=-4)=0.2+0.4]=0.6$	B1 **(1)**
(d)	$P(3X+2>5)=P(X>1)$ $P(3X+2>5)=P(X=3)+P(X=5)$ $P(3X+2>5)=0.35$	 M1 A1 **(2)**
(e)	$\text{Var}(aX+3)=a^2\,\text{Var}(X)$ $53.4=a^2 13.35$ $a=\pm 2$	M1 A1 **(2)** **[Total 9]**

	Notes
(a)	M1 for equating sum of all probabilities to 1 The minimum working required is: $0.6+2p=1$ but $2p=1-0.6$ or $2p=0.4$ is M0 BUT allow $1-0.4-0.05-0.15=0.4$ followed by $2p=0.4$ <u>or</u> $1-0.4-0.05-0.15=2p$ Since <u>all</u> of the probabilities are seen. A1cso for a correct solution with no incorrect working seen (For verify method, they must conclude that $p=0.2$)
(b)	M1 for a correct expression with at least 3 correct terms May be: $-1.6-0.4+0.05+0.45+1$ A1 for -0.5
(c)	B1 for 0.6
(d)	M1 for identifying $X=3$ and $X=5$ only ($X>1$ is not sufficient) A1 for 0.35
(e)	M1 for $\text{Var}(aX+3)=a^2\,\text{Var}(X)$ but this may be implied by seeing $a=2$ <u>or</u> $a=-2$ A1 for <u>both</u> correct values + 2 <u>and</u> -2

Q4.

Qu	Scheme	Marks	AO
(a)	$P(X=4)=P(X=2)$ so $P(X=4)=0.35$ $P(X=1)=P(X=3)$ and $P(X=1)+P(X=3)=1-0.7$ So x: 1, 2, 3, 4 $P(X=x)$: 0.15, 0.35, 0.15, [0.35]	M1 A1 (2)	2.1 1.1b
(b)	Let A = number of spins that land on 4 $A \sim B(60, "0.35")$	B1ft	3.3
	$[P(A>30)=]\ 1-P(A\leqslant 30)$ $=1-0.99411\ldots$ = awrt **0.00589**	M1 A1 (3)	3.4 1.1b
(c)	$Y-X\leqslant 4 \Rightarrow \frac{12}{X}-X\leqslant 4$ or $12-X^2\leqslant 4X$ (since $X>0$) o.e.	M1	3.1a
	i.e. $0\leqslant X^2+4X-12 \Rightarrow 0\leqslant (X+6)(X-2)$ so $X\geqslant 2$	M1	1.1b
	$P(Y-X\leqslant 4)=P(X\geqslant 2)=0.35+0.15+0.35$ = **0.85**	A1 (3)	3.2a
		(8 marks)	

Notes

(a)

M1 for using the given information to obtain $P(X=4)$
Award for statement $P(X=4)=P(X=2)$ or writing $P(X=4)=0.35$

A1 for getting fully correct distribution (any form that clearly identifies probs)
e.g. can be list $P(X=1)=0.15$, $P(X=3)=\ldots$etc or as a probability function $P(X=x)=\begin{cases}0.15 & x=1,3\\0.35 & x=2,4\end{cases}$
[Condone missing $P(X=2)$ as this is given in QP]

(b)

B1 for selecting a suitable model, sight of B(60, their 0.35) o.e. in words f.t. their $P(X=4)$ from part (a).
Can be implied by $P(A\leqslant 30)$ = awrt 0.9941 or final answer = awrt 0.00589

M1 for using their model and interpreting "more than half"
Need to see $1-P(A\leqslant 30)$. Can be implied by awrt 0.00589
Can ignore incorrect LHS such as $P(A\geqslant 30)$

A1 for awrt 0.00589

(c)

1st M1 for translating the prob. problem into a correct mathematical inequality
Just an inequality in 1 variable. May be inside a probability statement.

ALT **Table of values:**

X	1	2	3	4
Y	12	6	4	3

or values of $Y-X=11, 4, 1, -1$

2nd M1 for solving the inequality leading to a range of values, allow 1 or 2 slips
May be a quadratic or cubic but must lead to a set of values of X or $Y-X$

ALT **Table or values:** They must state clearly which values are required
Both Ms can be implied by a correct answer (or correct ft of their distb'n)

A1 for interpreting the inequality and solving the problem i.e. 0.85 cao

Q5.

	Scheme	Marks	AO
(a)	[Sum of probs = 1 implies] $\log_{36} a+\log_{36} b+\log_{36} c=1$	M1	3.1a
	$\Rightarrow \log_{36}(abc)=1$ so $abc=36$	A1	3.4
	All probabilities greater than 0 implies each of a, b and $c > 1$	B1	2.2a
	$36=2^2\times3^2$ (or 3 numbers that multiply to give 36 e.g. 2, 2, 9 etc)	dM1	2.1
	Since a, b and c are distinct must be **2, 3, 6** **($a = 2, b = 3, c = 6$)**	A1	3.2a
		(5)	
(b)	$(\log_{36} a)^2+(\log_{36} b)^2+(\log_{36} c)^2$	M1	3.4
	[= 0.0374137…+ 0.09398737…+0.25]		
	= 0.38140… awrt **0.381**	A1	1.1b
		(2)	
		(7 marks)	

	Notes
(a)	1st M1 for a start to the problem using sum of probabilities leading to eq'n in a, b and c
	1st A1 for reducing to the equation $abc = 36$ [Must follow from their equation.]
NB	Can go straight from $abc = 36$ to the answer for full marks for part (a).
	B1 for deducing that each value > 1 (may be implied by 3 integers all > 1 in the next line)
	2nd dM1 (dep on M1A1) for writing 36 as a product of prime factors or 3 values with product = 36 and none = 1
	2nd A1 for 2, 3 and 6 as a list or $a = 2$, $b = 3$ and $c = 6$
SC	**M0M0** If no method marks scored but a correct answer given score: M0A0B1M0A1 (2/5)
Ans only	This gets the SC score of 2/5 [Question says show your working clearly]
(b)	M1 for a correct expression in terms of a, b and c or values; ft their integers a, b and c Condone invisible brackets if the answer implies they are used.
	A1 for awrt 0.381