Binomial Hypothesis Testing Problems Exam Questions (From OCR 4766)

Q1 (Jun 2016, Q7)

To withdraw money from a cash machine, the user has to enter a 4-digit PIN (personal identification number). There are several thousand possible 4-digit PINs, but a survey found that 10% of cash machine users use the PIN '1234'.

(i) 16 cash machine users are selected at random.

(*A*) Find the probability that exactly 3 of them use 1234 as their PIN. **[3]**

(*B*) Find the probability that at least 3 of them use 1234 as their PIN. **[2]**

(*C*) Find the expected number of them who use 1234 as their PIN. **[1]**

An advertising campaign aims to reduce the number of people who use 1234 as their PIN. A hypothesis test is to be carried out to investigate whether the advertising campaign has been successful.

(ii) Write down suitable null and alternative hypotheses for the test. Give a reason for your choice of alternative hypothesis. **[4]**

(iii) A random sample of 20 cash machine users is selected.

(*A*) Explain why the test could not be carried out at the 10% significance level. **[3]**

(*B*) The test is to be carried out at the k% significance level. State the lowest integer value of k for which the test could result in the rejection of the null hypothesis. **[1]**

(iv) A new random sample of 60 cash machine users is selected. It is found that 2 of them use 1234 as their PIN. You are given that, if $X \sim \mathrm{B}(60, 0.1)$, then (to 4 decimal places)

$$\mathrm{P}(X = 2) = 0.0393, \qquad \mathrm{P}(X < 2) = 0.0138, \qquad \mathrm{P}(X \leqslant 2) = 0.0530.$$

Using the same hypotheses as in part **(ii)**, carry out the test at the 5% significance level. **[4]**

Q2 (Jun 2015, Q7)

A drug for treating a particular minor illness cures, on average, 78% of patients. Twenty people with this minor illness are selected at random and treated with the drug.

(i) (*A*) Find the probability that exactly 19 patients are cured. **[3]**

(*B*) Find the probability that at most 18 patients are cured. **[3]**

(*C*) Find the expected number of patients who are cured. **[1]**

(ii) A pharmaceutical company is trialling a new drug to treat this illness. Researchers at the company hope that a higher percentage of patients will be cured when given this new drug. Twenty patients are selected at random, and given the new drug. Of these, 19 are cured. Carry out a hypothesis test at the 1% significance level to investigate whether there is any evidence to suggest that the new drug is more effective than the old one. **[8]**

(iii) If the researchers had chosen to carry out the hypothesis test at the 5% significance level, what would the result have been? Justify your answer. **[2]**

Q3 (Jun 2014, Q7)

It is known that on average 85% of seeds of a particular variety of tomato will germinate. Ramesh selects 15 of these seeds at random and sows them.

(i) (A) Find the probability that exactly 12 germinate. **[3]**

(B) Find the probability that fewer than 12 germinate. **[2]**

The following year Ramesh finds that he still has many seeds left. Because the seeds are now one year old, he suspects that the germination rate will be lower. He conducts a trial by randomly selecting n of these seeds and sowing them. He then carries out a hypothesis test at the 1% significance level to investigate whether he is correct.

(ii) Write down suitable null and alternative hypotheses for the test. Give a reason for your choice of alternative hypothesis. **[4]**

(iii) In a trial with $n = 20$, Ramesh finds that 13 seeds germinate. Carry out the test. **[4]**

(iv) Suppose instead that Ramesh conducts the trial with $n = 50$, and finds that 33 seeds germinate. Given that the critical value for the test in this case is 35, complete the test. **[3]**

(v) If n is small, there is no point in carrying out the test at the 1% significance level, as the null hypothesis cannot be rejected however many seeds germinate. Find the least value of n for which the null hypothesis can be rejected, quoting appropriate probabilities to justify your answer. **[3]**

Q4 (Jan 2013, Q7)

A coffee shop provides free internet access for its customers. It is known that the probability that a randomly selected customer is accessing the internet is 0.35, independently of all other customers.

(i) 10 customers are selected at random.

(A) Find the probability that exactly 5 of them are accessing the internet. **[3]**

(B) Find the probability that at least 5 of them are accessing the internet. **[2]**

(C) Find the expected number of these customers who are accessing the internet. **[2]**

Another coffee shop also provides free internet access. It is suspected that the probability that a randomly selected customer at this coffee shop is accessing the internet may be different from 0.35. A random sample of 20 customers at this coffee shop is selected. Of these, 10 are accessing the internet.

(ii) Carry out a hypothesis test at the 5% significance level to investigate whether the probability for this coffee shop is different from 0.35. Give a reason for your choice of alternative hypothesis. **[9]**

(iii) To get a more reliable result, a much larger random sample of 200 customers is selected over a period of time, and another hypothesis test is carried out. You are given that 90 of the 200 customers were accessing the internet. You are also given that, if X has the binomial distribution with parameters $n = 200$ and $p = 0.35$, then $\mathrm{P}(X \geqslant 90) = 0.0022$. Using the same hypotheses and significance level which you used in part **(ii)**, complete this test. **[2]**

Q5, (Jun 2013 Q5)

A researcher is investigating whether people can identify whether a glass of water they are given is bottled water or tap water. She suspects that people do no better than they would by guessing. Twenty people are selected at random; thirteen make a correct identification. She carries out a hypothesis test.

(i) Explain why the null hypothesis should be $p = 0.5$, where p represents the probability that a randomly selected person makes a correct identification. **[2]**

(ii) Briefly explain why she uses an alternative hypothesis of $p > 0.5$. **[1]**

(iii) Complete the test at the 5% significance level. **[5]**

Q6 (Jun 2012, Q5)

A manufacturer produces titanium bicycle frames. The bicycle frames are tested before use and on average 5% of them are found to be faulty. A cheaper manufacturing process is introduced and the manufacturer wishes to check whether the proportion of faulty bicycle frames has increased. A random sample of 18 bicycle frames is selected and it is found that 4 of them are faulty. Carry out a hypothesis test at the 5% significance level to investigate whether the proportion of faulty bicycle frames has increased. **[8]**
